Morphological and Statistical Techniques for the Analysis of 3D Images

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3–3–2011
Outline

Motivation: 2D/3D differences

The 3D tree of shapes

Applications of the tree of shapes

Finsler-Cheeger Sets and Affine Invariants

Conclusion
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Motivation: differences between 2D and 3D

A large part of this thesis is the extension of the tree of shapes to 3D images, and its applications.

Differences 2D/3D: semantics, visualization, topology, statistics, noise

- photo \((x, y)\)
- tomography \((x, y, z)\)
- video \((x, y, t)\)
- X-ray \((x, y)\)
- RGB histogram \((r, g, b)\)
- scale-space \((x, y, s)\)
Different visualization of 2D and 3D images

The first difference is the need for visualization.

- 2D, trivial: simply look at them (array of pixels)
- 3D, tricky: slices, projections, isosurfaces, volume rendering
Topological differences between 2D and 3D

The second difference is topological behaviour.

- 2D: cavities and handles are the same thing
- 3D: cavities and handles are different things

Due to this fact, for example, the number of cavities can be computed locally in 2D but not in 3D.
Statistical differences between 2D and 3D

Critical levels for bound percolation:
- 2D grid: 0.5
- 3D grid: 0.2488

This affects, for example, the behaviour of level sets of white noise (at the mean value):
- in 2D: many connected components
- in 3D: a single connected component
Different kinds of noise between 2D and 3D images

- 2D: noise from cameras is independent from pixel to pixel
- 3D: noise from CT has anisotropic artifacts

Slice of a tomography
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There are many data structures for storing digital images:

- Array of pixel values
- Coefficients on a linear basis (e.g., Fourier representation)
- Tree-like data structures:
  - Quadtrees/octtrees [Samet'79, Meagher'82]
  - Region merging trees [Koepfler et al. '94, Salembier et al. '00]
  - Tree of (upper or lower) level sets [Cox et al. '03, Carr et al. '03]
  - Tree of shapes [Monasse et al. '02]

Each representation favors some operations.
Mathematical definition of the tree of shapes

**Intuition:** Shapes = connected components of level surfaces
= external boundaries of saturations

**Notation:**
- Image $f : \Omega \to \mathbb{R}$
- $[f \geq \lambda] := \{x \in \Omega : f(x) \geq \lambda\}$
- $cc(A, x) :=$ connected component of $A$ containing $x$
- $Sat(A, p_\infty) := \Omega \setminus cc(\Omega \setminus A, p_\infty)$

**Trees of level sets:**
- $\mathcal{ULT}(f) := \{cc([f(x) \geq \lambda], x) : x \in \Omega, \lambda \in \mathbb{R}\}$
- $\mathcal{LLT}(f) := \{cc([f(x) < \lambda], x) : x \in \Omega, \lambda \in \mathbb{R}\}$

**Tree of shapes:**
- $\mathcal{TOS}(f) := \{Sat(r), r \in \mathcal{ULT}(f) \cup \mathcal{LLT}(f)\}$
Tree of shapes as fusion of upper and lower trees

LLT

ULT

image
Tree of shapes as fusion of upper and lower trees

break branches

glue branches

$TOS$
Implementation

Discretization of the image domain, two choices:

- semi-continuous interpolation
- graph-based

Algorithms:
- Building the upper tree: pixel merging
- Joining both trees: branch gluing
- Where to glue: find the smallest “outer adjacent” region
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Applications of the 3D tree of shapes

- Morphological operations (tree pruning)
- Image visualization (tree navigation)
- RGB histogram analysis
- Video analysis
- 3D edge detection
Morphological filtering

Grain filtering: removal of small leaves

original image  
opening  
closing  
grain filter

original image  
18351 shapes  
t=2  
11173 (61%)  
t=10  
4924 (27%)  
t=1000  
715 (4%)
Morphological filtering

Grain filtering: removal of small leaves

original image
295937 shapes

2 t=2
141005 (47%)

t=100
2414 (0.8%)

t=10000
124 (0.04%)

Observation: it does not seem to affect the texture.
Morphological filtering

Adaptive quantization: removal of nodes inside long branches
Image visualization

Navigating the tree (with keyboard or mouse):
RGB histogram analysis

gray image

[Image]

color image

visualizations of its 3D histogram
RGB histogram analysis
RGB histogram analysis
Video analysis

1. Build the structure
   - Input: video & optical flow
   - Pre-processing: graph of optical flow connectivity
   - Structure: level sets of video data on the graph

2. Work with it:
   - Grain filtering: remove spots, small objects
   - Select a region: tracking
   - Automatic segmentation from scribbles
   - Monocular depth estimation
Video processing using the tree of shapes

Grain filtering and branch simplification turn the video into a small set of “tubes”

input video

$2s \approx 6.7 \cdot 10^6$ voxels

simplified tree of shapes

617 shapes
Tracking using the tree of shapes

Tracking means querying the structure:

one shape of a video sequence
Monocular depth estimation

The discontinuities of the flow along occlusions are a monocular depth cue.
Monocular depth estimation

(monocular depth estimation video)
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation
Monocular depth estimation

13/14
Monocular depth estimation
3D Edge detection

Extend to 3D images the 2D detector based on Helmholtz principle. [Desolneux et al. ’01]

► In 2D: *select* connected pieces of level curves which are well-contrasted
► In 3D: *select* connected pieces of level surfaces which are well-contrasted

✓ No priors
✓ No parameters
✗ Connected subsets of a given surface can be very complicated!
Mumford-Shah segmentation on surfaces

Solution: restrict the selection to the Mumford-Shah hierarchy of the “contrast” on each level surface.

- contrast on level surface
- best subset among all
- best subset on MS hierarchy
3D edge detection by Helmholtz principle

**Algorithm:**
* For each level surface $S$ of the image:
  * Build a Mumford-Shah hierarchy of the contrast on $S$
  * For each patch $P$ in the hierarchy
    * if $P$ is well-contrasted, select $P$
Hypothesis of the proposed edge detector

The desired boundaries are formed by large pieces of level surface of the original image.

Observation: this hypothesis is more sound for 3D medical images than for 2D photographs.
Experimental results of edge detection

Synthetic best case:

input

output
(3 patches)

Canny
Experimental results of edge detection

Real CTA image:

input

output
(11 patches)

Canny
Experimental results of edge detection

Real MRI image:

input

output
(17 patches)

isosurface
Pre and post-processing of the edge detector

The performance of the method can be improved in several ways

Pre-processing:

- Grain filter (reduces the number of small shapes)
- Median filtering (reduces the complexity of noisy shapes)
- Quantization (reduces the number of nested shapes)
- Canny instead of gradient norm (improves localization)

Post-processing:

- Exclusion principle (allow only one output patch per voxel)
- Surface joining (join overlapping patches into one)
Surface joining (or 3D edge linking)

Some functionals for edge linking data $S$ with model $\Gamma$:

- Osher-Zhao: $E_0(\Gamma) = \int_{\Gamma} ds$
- “reverse Osher-Zhao”: $E_1(\Gamma) = \int_S |d\Gamma|$
- Adjustment of distance functions: $E_2(\Gamma) = \int_{\mathbb{R}^3} |ds - |d\Gamma||$
- Combinations of the above ($E_0 + E_1$, etc.)
Surface joining (Osher-Zhao)

input patches

joined surface
Surface joining (Osher-Zhao)

input patches

joined surface
Trivial global minima

Problem: all the functionals above have trivial global minima

Solution: adding an “inflating force” to the functional may improve the situation.

\[ E(\Gamma) = E_i(\Gamma) - \lambda \cdot \text{volume}(\Gamma) \]
Edge linking with an inflating force can be formulated in terms of anisotropic Cheeger sets.

Let us forget about edge linking, and focus on Finsler-Cheeger sets.
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Finsler metrics in image processing

Finsler (or Riemannian) metrics are widely used in image processing, and elsewhere.

geodesic active contours, anisotropic diffusion...
Euclidean Cheeger sets

- Cheeger constant of $\Omega$: $C_\Omega := \min_{A \subseteq \Omega} \frac{|\partial A|}{|A|}$
- Cheeger set of $\Omega$: set $A$ where the minimum is attained
- Equivalent definition: minimizer of $|\partial A| - C_\Omega|A|$

Images from http://www.ceremade.dauphine.fr/~peyre/cheeger/

let us move to Finsler spaces
Finsler-Cheeger sets

Let us weigh perimeters by $\varphi$ and volumes by $h$.

- Cheeger constant of $\Omega$: $C_\Omega := \min_{A \subseteq \Omega} \frac{|\partial A|_\varphi}{|A|_h}$
- Cheeger set of $\Omega$: set $A$ where the minimum is attained
- Equivalent definition: minimizer of $|\partial A|_\varphi - C_\Omega |A|_h$
Finsler-Cheeger sets

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- Cheeger set of $\Omega$: set $A$ where the minimum is attained
- Equivalent definition: minimizer of $|\partial A|_\varphi - C_\Omega |A|_h$

How to compute these objects?
Cheeger sets and total variation

Euclidean case:

- Variational problem:
  \[ E_\lambda(u) := \int_\Omega |Du| + \frac{\lambda}{2} \int_\Omega (u - 1)^2 + \int_{\partial\Omega} |u| \]
- Cheeger sets arise as level sets of the solution: \( u \geq |u|_\infty \)

Finslerian case:

- Variational problem:
  \[ E_{\varphi,h,\lambda}(u) := \int_\Omega |Du|_\varphi + \frac{\lambda}{2} \int_\Omega (u - 1)^2 h + \int_{\partial\Omega} \varphi(x, \nu_{\Omega}) |u| \]
- \( \varphi \)-Cheeger sets arise as level sets of the solution: \( u \geq |u|_\infty \)
Numerical solution of the variational problem

- Euclidean case: Chambolle algorithm

\[ p^{t+1} = \frac{p^n + \tau \nabla \text{div}(p^n)}{1 + \tau |\nabla \text{div}(p^n)|} \]

- Finsler case: Chambolle algorithm with suitable weights

\[ p^{t+1} = \frac{p^n + \tau A_x \cdot \nabla \left( h^{-1} \text{div}(A_x \cdot p^n) \right)}{1 + \tau |A_x \cdot \nabla \left( h^{-1} \text{div}(A_x \cdot p^n) \right)|} \]

Once these iterations converge, the Cheeger set is found among the level sets of the resulting image \( \text{div}(p) \).
Numerical computation of the Cheeger set

We recover our old friend, the tree of shapes.

- The Cheeger ratio is defined for every shape
- Find a minimum of the Cheeger ratio defined on the tree of shapes (which is a topological space)

Euclidean Cheeger set of a cube
Computation of Finsler-Cheeger ratio

Finsler-Cheeger ratio numeric/analytic
Applications of Finsler-Cheeger sets

By fine-tuning the Finsler metric, we can make the Cheeger sets to have any shape that we want.

(For example, by letting the metric vanish on a given curve)
**Application to edge linking**

The $\varphi$-Cheeger is a global optimum of Osher-Zhao with an inflating force

"Distance Cheegers": $\varphi = \text{distance to data}
Application to edge linking

\( d_S \)-Cheeger set of the whole image domain
Application to active contours

The $\varphi$-Cheeger is a global optimum of active contours with inflating force

“Gradient Cheegers”: $\varphi = \frac{1}{|\nabla I|}$
Application to active contours

The $\varphi$-Cheeger is a global optimum of active contours with inflating force

$\frac{1}{|\nabla u|}$-Cheeger set of the whole image domain

$\frac{1}{|\nabla u|}$-Cheeger set of the image domain minus some manually selected voxels
Finsler-Cheeger sets and MSER

There is a connection between $\frac{1}{|\nabla u|}$-Cheegers and MSER.

MSER [Matas et al. '04] are defined as minimizers of

$$F(\lambda) := \frac{|[u < \lambda + \delta]| - |[u < \lambda - \delta]|}{|[u < \lambda]|}$$

By co-area formula,

$$F(\lambda) = 2\delta \int_{\{|u|=\lambda\}} \frac{1}{|\nabla u|} d\mathcal{H}^{N-1}$$

Which is almost exactly the $\frac{1}{|\nabla u|}$-Cheeger ratio!

MSER can be regarded as a crude approximation to (local) $\frac{1}{|\nabla u|}$-Cheegers
Application to logo detection

Match MSER and other features between a logo database and a video frame.
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Contributions of this thesis

- Algorithm for 3D tree of shapes
- Basic applications (grain filters, visualization, RGB histograms, video analysis)
- 3D edge detection (exclusion principle, MS hierarchies, consistent Marching Cubes)
- Numerical computation of Finsler-Cheeger sets
- Application of Finsler-Cheeger sets to edge linking
- MSER are local Finsler-Cheeger sets
Future work

- User interface for tree of shapes navigation and editing
- Joining the trees of level sets defined on a graph
- Quantitative comparison of edge detectors
- Quantitative comparison of surface joiners
- Development of object detection based on the new affine invariants
- Faster schemes for Finsler-Cheeger sets
Publications

Published:

- **Constructing the Tree of Shapes of an Image by Fusion of the Trees of Connected Components of Upper and Lower Level Sets**
  V. Caselles, E. Meinhardt and P. Monasse

- **Edge Detection by Selection of Pieces of Level Lines**
  E. Meinhardt
  *International Conference on Image Processing 2008*

- **3D Edge Detection By Selection of Level Surface Patches**
  E. Meinhardt, E. Zacur, A.F. Frangi and V. Caselles
  *Journal of Mathematical Imaging and Vision*,
  Vol. 32, Num. 1, pp. 1–16, 2009

- **Anisotropic Cheeger Sets and Applications**
  V. Caselles, G. Facciolo and E. Meinhardt
  *SIAM Journal on Imaging Sciences*,
  Vol. 2, Num. 4, pp. 1211–1254, 2009

- **A Robust Pipeline for Logo Detection**
  E. Meinhardt, C. Constantinopoulos, V. Caselles
  *International Conference on Multimedia and Expo 2011*

Under review:

- **Relative Depth Estimation from Monocular Video**
  E. Meinhardt, V. Caselles
  Submitted to *International Conference on Image Processing 2011*

- **On Affine Invariant Descriptors Related to SIFT**
  R. Sadek, C. Constantinopoulos, E. Meinhardt, C. Ballester, V. Caselles
  Submitted to *SIAM Journal on Imaging Sciences on 2010*