Three-dimensional edge detection by Helmholtz principle

Enric Meinhardt-Llopis[1], *Co-authors*: Ernesto Zacur[2], Vicent Caselles[1], Alejandro Frangi[1].

[1]Universitat Pompeu Fabra, Barcelona; [2]Universidad de Zaragoza

Abstract

We propose a new edge detector for three-dimensional gray-scale images, based on the two-dimensional edge detector of Desolneux *et.al.* [1]. While the edges of a planar image are pieces of curve, the edges of a spatial image are portions of surface, that are more delicate to manage. The proposed edge detector works by selecting those portions of level surface which are well-contrasted according to a statistical test, called Helmholtz principle. As it is infeasible to treat all the possible portions of each level surface, we restrict the search to the regions that appear in the Mumford-Shah segmentation of the gradient over the surface, through all the scales. We assert that this selection device results in a good edge detector for a wide class of images, including several types of medical images from tomography and magnetic resonance.

Background

The problem of edge detection

Edge detection is the task of finding the boundaries of the objects that appear in a digital image. Segmentation is a different, but closely related problem, that consists in finding the objects themselves. Both problems have different constraints and applications. Edge detection, being of a lower level nature than segmentation, favors picking structures all over the image, and needs no initialization.

Typical edge detectors in 2D

Two-dimensional edge detectors can be classified into two types, according to the kind of output they produce.

Point-wise methods

Those are usually differential operators that measure the "edgeness" of every point in the image. Two well known examples of this are the contrast (norm of the gradient) $\|\nabla u\|$ and the zero crossings of Canny's operator $Hu(\nabla u, \nabla u)$.

- These operators are easy to compute
- The are trivially generalizable to 3D
- They give a set of disconnected points, without structure, which is difficult to use

Curve-based methods

Curve-based edge detectors produce a set curves as their output. They can work by "edge-linking" the output from a point-wise method; or directly on curves, such as the detector of Desolneux *et.al.*, which we generalize here.

- These methods give better results, which are readily usable in applications (such as matching shapes between images).
- They are not trivially generalizable to 3D, because surfaces are much more complex than curves.

Our method

Input

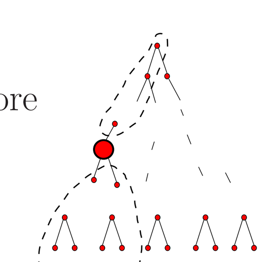
- The original gray-scale image
- The image of contrast (norm of the gradient of the original image)
- A sensitivity parameter ϵ

Output

- A set of pieces of surface

Algorithm

- For each connected component S of each level surface of the grayscale image:
  - Interpolate the contrast at the vertices of the triangulated surface S
  - Compute the Mumford-Shah tree T of the contrast on S
  - Run the statistical test (with parameters $(N^*, \min, \epsilon)$) to all the nodes of T
  - While there are still nodes in T:
    - Pick the node n of T that passes the statistical test with highest score
    - Output the corresponding piece of surface
    - Remove from T the node n and all its ancestors and descendants

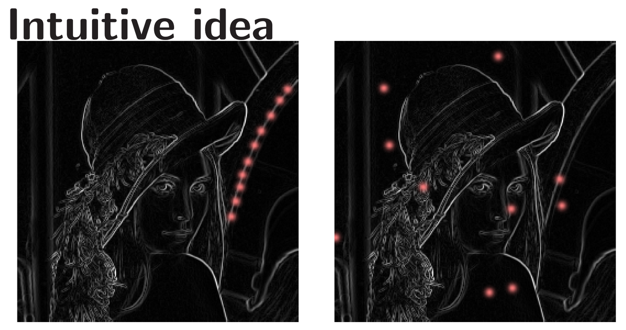


*Note: N is an estimation of the total number of surface patches, such as $N = \sum_S 2 * \text{area}(S)$.

Well-contrasted subsets of an image

Let us define a statistical test to decide whether sets of points in an image are well-contrasted or not.

Intuitive idea



These figures display two different sets of 10 points thrown in the lena image. We want the first subset to pass the test and the second subset to fail it.

Formal model

Data (a-contrario model)

- A collection of N sets of i.i.d. random variables $S_i = \{X_i^1, \dots, X_i^{n_i}\}$, $i = 1, \dots, N$
- The distribution of X_1, given e.g. by its forward cumulative distribution $H(\mu) = \mathbb{P}(X_1 \geq \mu)$

Parameters

- A symmetric and increasing statistic $f(Y_1, \dots, Y_n)$, e.g. $f = \min$ or $f = \text{mean}$
- A sensitivity parameter $\epsilon > 0$

Test

- Compute the distribution $\tilde{f}$ of f:
- The i-th set passes the test when $N\tilde{f}(f(X_i^1, \dots, X_i^{n_i})) < \epsilon$

Property (providing an interpretation of the test)

The expectation of the number of sets that pass the test is bounded by ϵ.

Use of the model

We can use the model above to decide which subsets of an image (within a given collection of size N) are significantly well-contrasted. We assume that the values of the contrast are i.i.d., their distribution being approximated by the histogram of the gradient. Then we run the test with $f = \min$ and $\epsilon = 0.1$. This means that a set of n points and minimum contrast μ passes the test whenever $NH(\mu)^n < 0.1$. By the property above, the expected number of sets that pass the test is less than 0.1. If we find plenty of them, this means that the assumption of independence was not right, and the set deserves a different explanation. This methodology is called *Helmholtz principle* and is used widely in Gestalt theory.

Mumford-Shah segmentation on surfaces

General setting

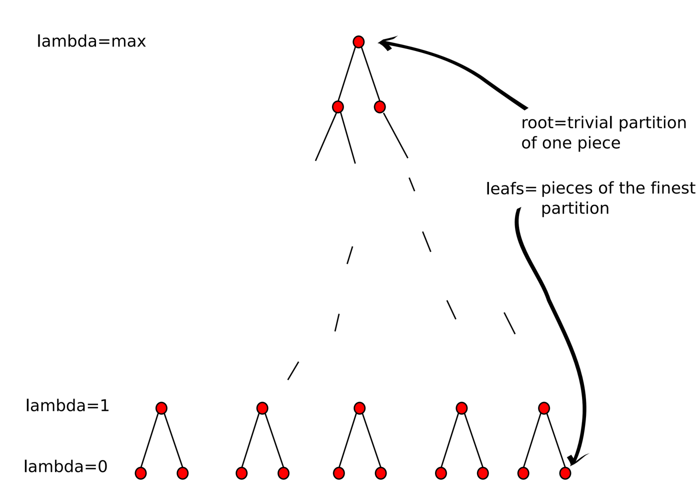
Let Ω be a Riemannian manifold. The **piecewise constant Mumford-Shah segmentation** [2] of a function $I : \Omega \to \mathbb{R}$ is defined by the partition $\{\Omega_i\}$ of Ω that minimizes the following energy:

$$\sum_i \text{Variance of } I \text{ on } \Omega_i + \lambda \sum_{i,j} \text{Measure of border between } \Omega_i \text{ and } \Omega_j$$

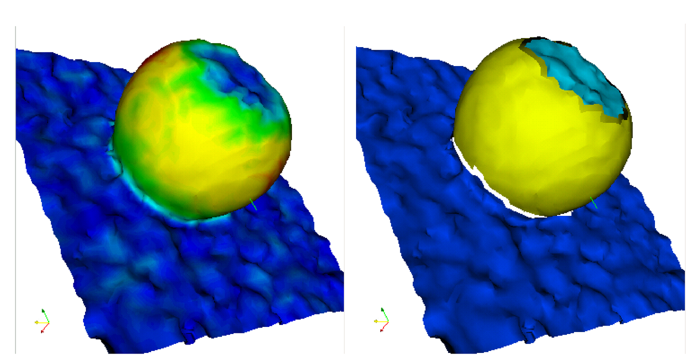
and an approximation is obtained assigning to each region the mean value of I over it. The positive number λ is a scale parameter.

Tree of mergings

We can (approximately) organize the segmentations given by all values of λ into a tree. Each node of the tree corresponds to some region of Ω. The leafs of the tree come from an initial over-segmentation, and the root of the tree is the whole set Ω.



Use of the tree of mergings



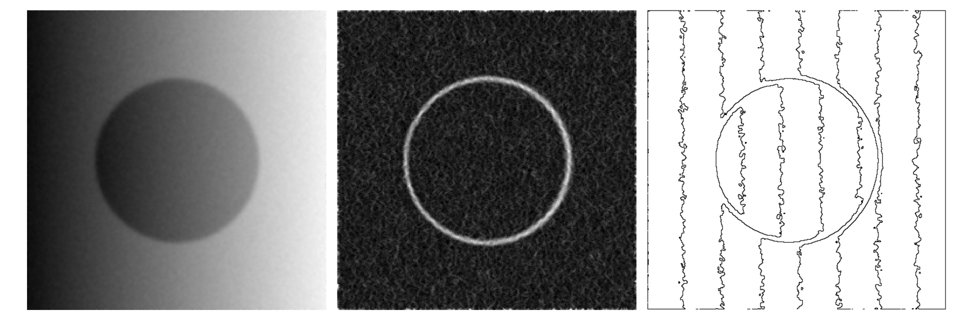
We can take Ω to be a connected component of a level surface of an image, and I to be the contrast evaluated at its points. Then the tree described above **provides a collection of patches of Ω of all possible sizes**, where each patch has a more or less uniform contrast. Here we show the contrast over one level surface and a Mumford-Shah segmentation with a λ such that there are 3 regions.

Applicability of our hypotheses

Our method relies on the following two hypotheses:

- The boundaries are formed by large pieces of level surface of the original gray-scale image
- The boundaries have as high contrast as possible

Or, in other words, that **objects are of different color** and that **the boundary is located where the color varies more rapidly**. Whether this hypotheses apply or not depends on the kind of image. For example, they are true in the images displayed on the next section, while it fails in images where the objects are defined by textures (e.g., ultrasound).



Synthetic image: Slice of the image, its gradient, and some of its level lines.

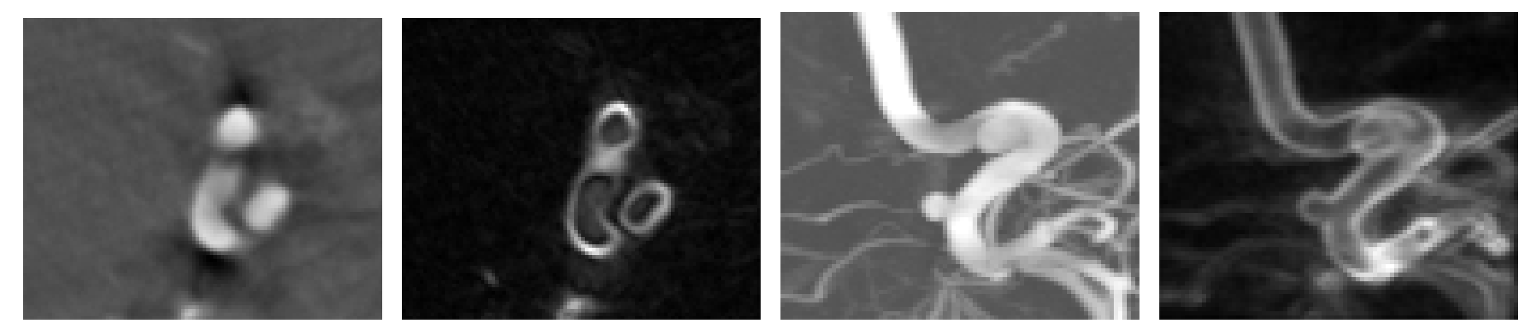
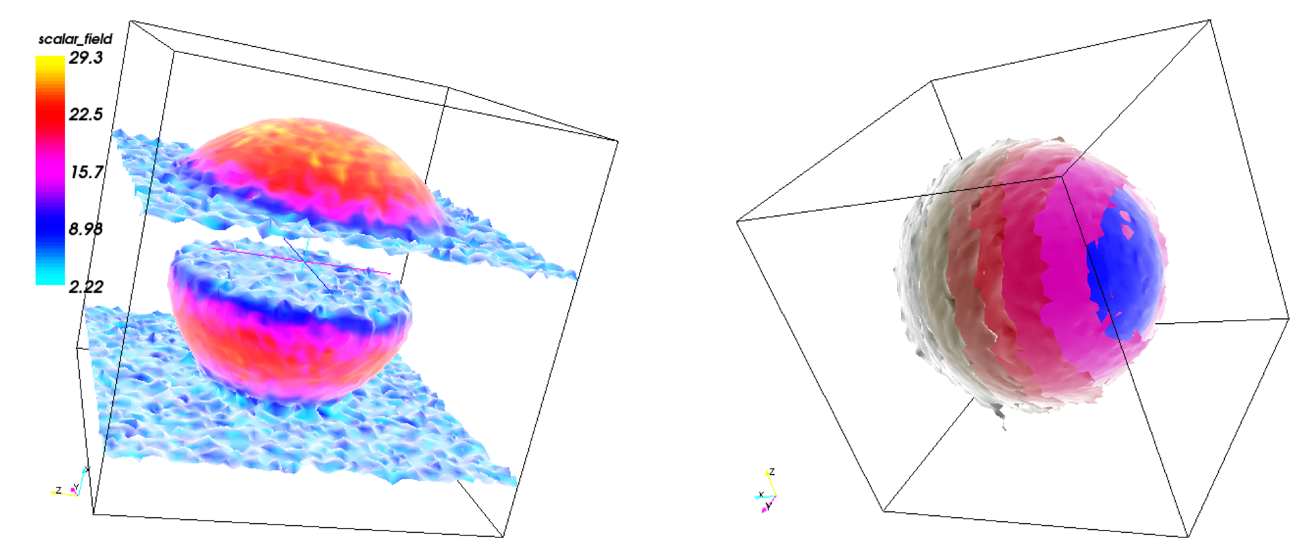


Image of aneurysm: Slice and projection of the original gray-scale image and of its contrast.

Sample results

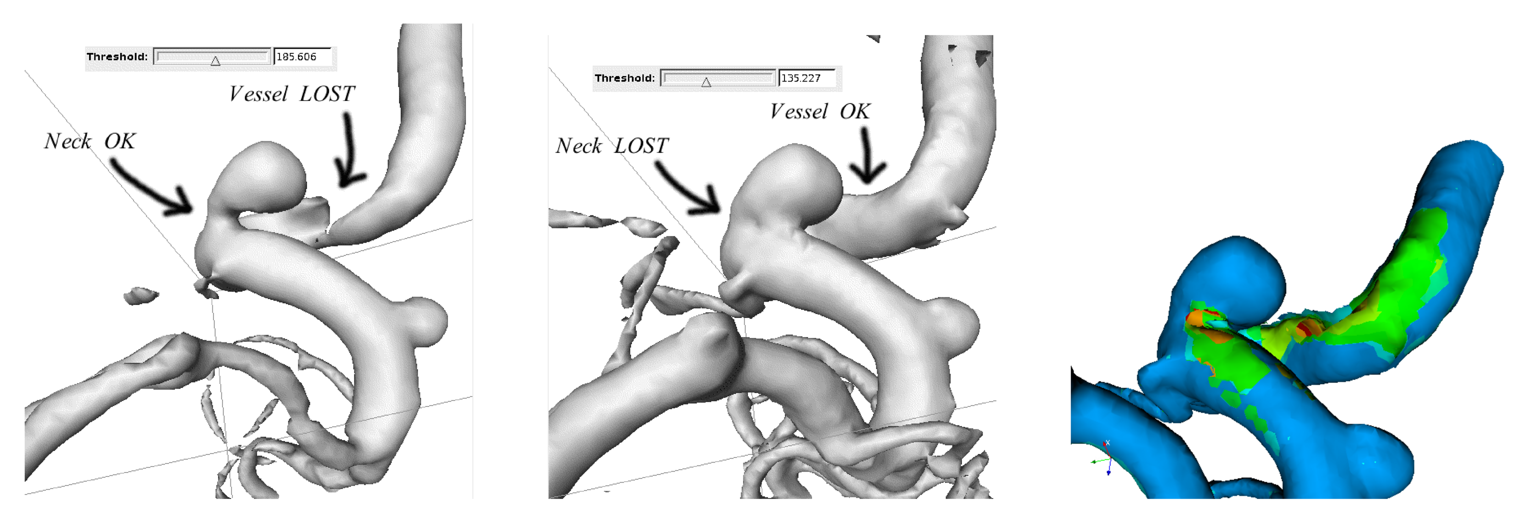
We display the results of our algorithm for both a synthetic image and a real image of an aneurysm.

Synthetic image



Left: two level surfaces of the image, where the color represents the contrast. **Right:** Output of our edge detector, where the color is a labeling of the pieces.

Aneurysm in CTA



Left and Center: two level surfaces of the image, showing different parts where thresholding fails to give correct segmentations. **Right:** Output of our edge detector.

[1] I. Desolneux, L. Moisan, and J. Morel. Edge detection by helmholtz principle. *Journal of Mathematical Imaging and Vision*, 2002.

[2] D. Mumford and J. Shah. *Optimal Approximations by Piecewise Smooth Functions and Associated Variational Problems*. Center for Intelligent Control Systems, 1988.