GEODESIC NEIGHBORHOODS FOR PIECEWISE AFFINE INTERPOLATION OF SPARSE DATA

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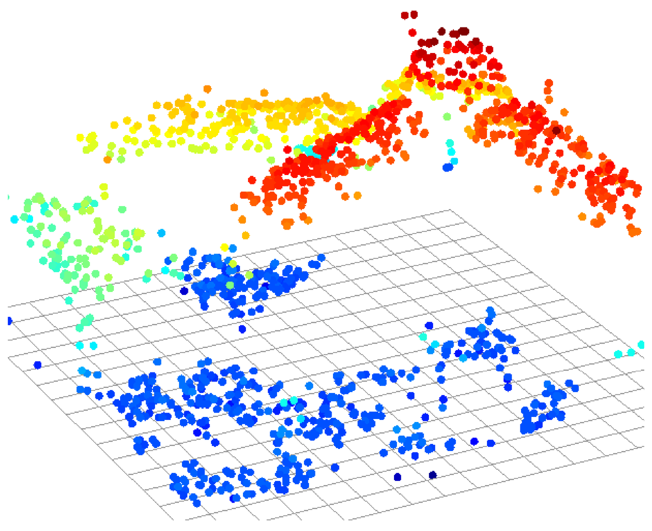
Abstract

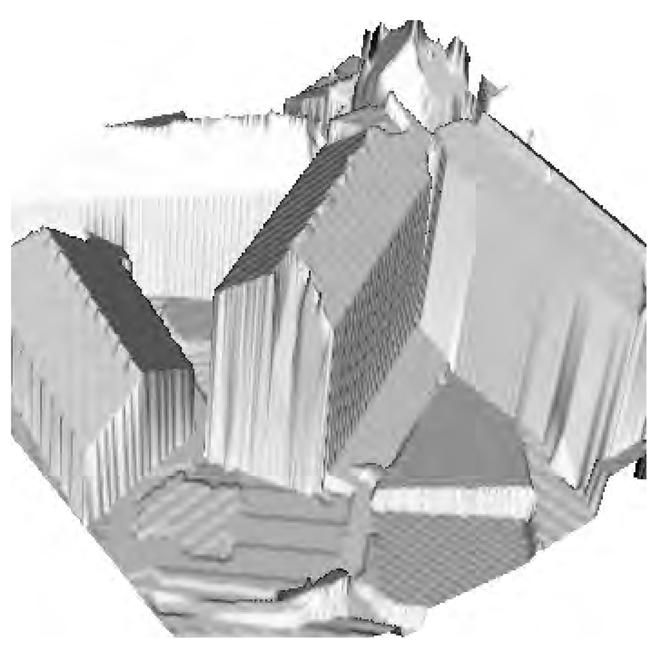
We propose a new interpolation method for sparse data that allows incorporation of geometric information of a reference image u. The idea consists in defining a geodesic Voronoi cell for each data sample, and fit a model to interpolate inside each cell.
A geodesic distance permits both: to effectively adapt the shape of the cells to the image structures; and to compute a set of neighboring samples that are used for fitting a piecewise affine model at each cell.

Objective

Interpolate a set of range measurements (LIDAR or sparse disparity data [1]) using the additional knowledge provided by a photograph u of the scene.



Samples with known depth (5% of the total pixels).



Interpolated samples.

Reference image u.

Lambertian hypothesis: *a uniform surface with a constant angle has a constant intensity in the image.*

Allows to extrapolate information across uniform regions of the image.

Geodesic Voronoi Cells and Neighborhoods

We use the geodesic distance to incorporate the radiometric information provided by the image u into the interpolation as in [2].

Reference Image $u(x):\Omega\to\mathbb{R}^+$, with $\Omega\subset\mathbb{R}^2$.

Positions of the samples $\Lambda\subset\Omega$.

Depth values of samples $G(\lambda):\Lambda\to\mathbb{R}$, $\lambda\in\Lambda$.

Curve $C(p)\ :\ [0,1]\ \to\ \Omega$, and $C_{s,t}$ curve connecting s and t.

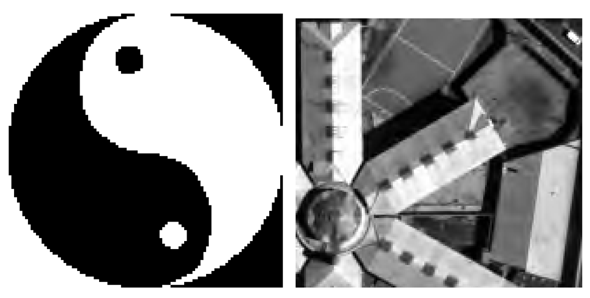
Geodesic distance between s and t measures the minimum variation of u between s and t

$$d(s,t)=\min_{C_{s,t}}\int_0^1 |\nabla u\cdot\dot{C}_{s,t}(p)|+\varepsilon|\dot{C}_{s,t}(p)|dp$$

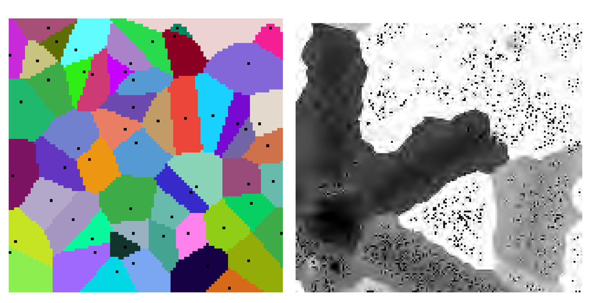
Observe: The shortest path is the one with less discontinuities of u along it.

Geodesic Voronoi diagram of the sites in Λ, successfully accounts for discontinuities in the image.

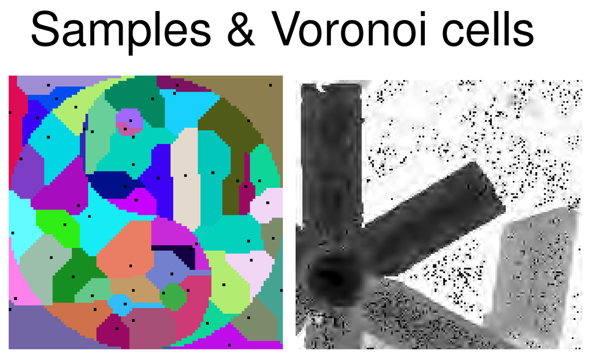
Geodesic neighborhood $GN_K(p)$ of the point p, is the set formed by the K-nearest (in the geodesic sense) samples of Λ to the point p. *Samples in* $GN_K(p)$ *are likely to have the same model as* p.



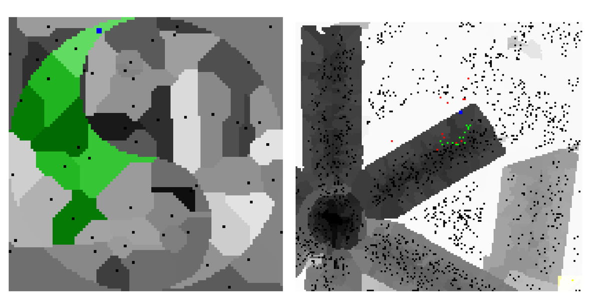
Reference image



Samples & Voronoi cells

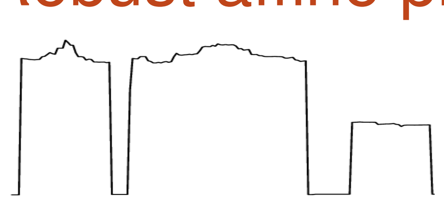


Geodesic Voronoi cells

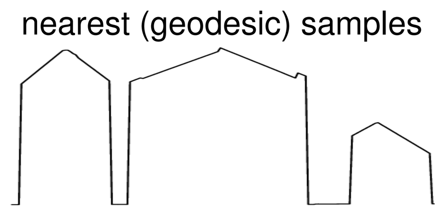


Geodesic Neighborhood

Robust affine plane interpolation using GN_K



Profile: Linear blend of the 5 nearest (geodesic) samples



Profile: Affine plane estim. using the same 5 samples

For each point $p\in\Lambda$ we fit an affine plane trough $GN_K(p)$. Then we extended the plane to the entire cell, **the initial piecewise affine model** H is the union of all the cells.

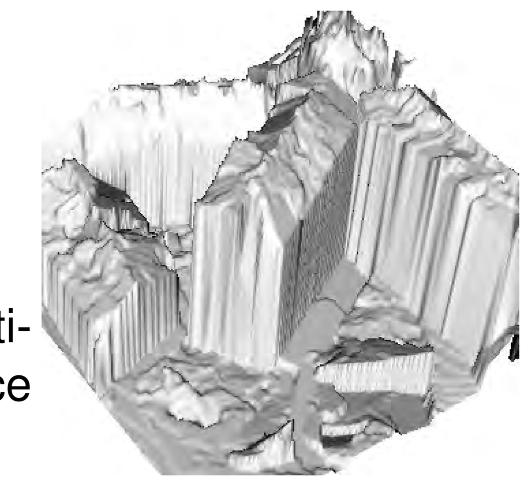
If $GN_K(p)$ contains outlier samples (that do not belong to the same surface as p) then the result will be biased. To remove these outliers we use a modified *RANdom SAmple Consensus* (RANSAC) [4].

Constrained region merging

Problem: When interpolating noisy data

- the model of each cell is independently estimated,
- adjacent cells may end up with different models.

Our solution: Merge adjacent regions with compatible models, and compute a common model to reduce noise effects.



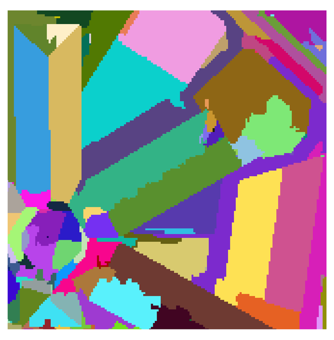
Result without merging

Before merging

Simplified Mumford-Shah [5] minimizes

$$E(B,f)=\sum_{R\in\mathcal{P}(\Omega)} RErr(R,f)+\lambda\int_B g(s)ds,\quad \lambda\geq 0.$$

- **Error term:** $RErr(X,f)=\sum_{x\in X\cap\Lambda}|f_X(x)-H(x)|^2$
  H: the initial plane interpolation
  f_X: the affine model for the region X
- **Boundary lenght:** $g(s)$ is big at poorly contrasted boundaries and small at well contrasted ones.
- Also constrained by the segments of the original image [6].
- Greedy algorithm: merges pair of regions while the error term is small.



After merging

Results

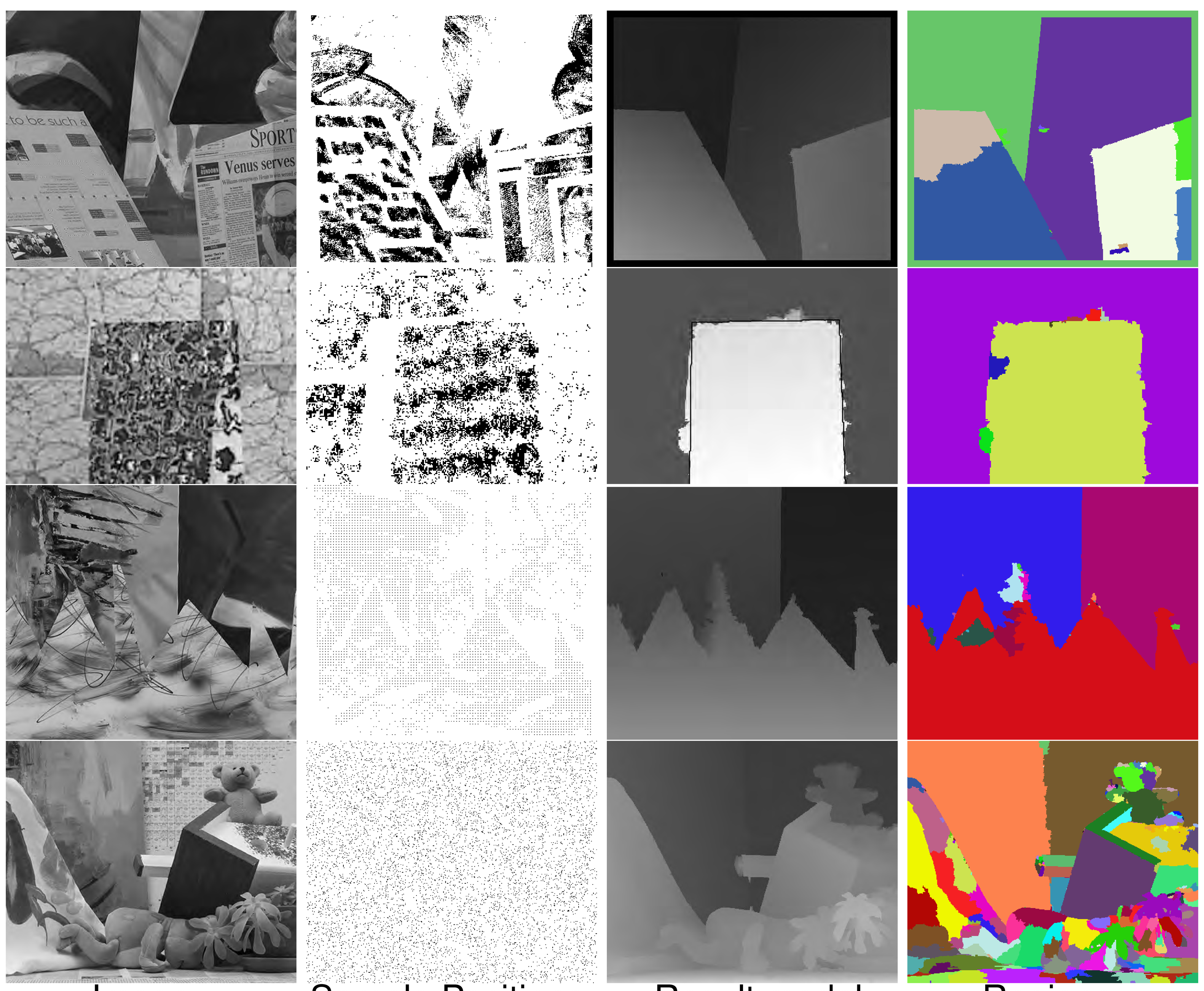


Image | Sample Positions | Result model | Regions

Discussion

- Method for interpolating range data that incorporates the geometric information provided by an image of the scene.
- Geodesic neighborhoods constitute a fast and robust tool for modelling the local information, it can be adapted to other (non-affine) models [3].
- Poor results for badly contrasted edges between strongly textured regions.

More results at: **http://gpi.upf.edu/static/geoint.**

References

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