Session 4: Muestreo y Alias

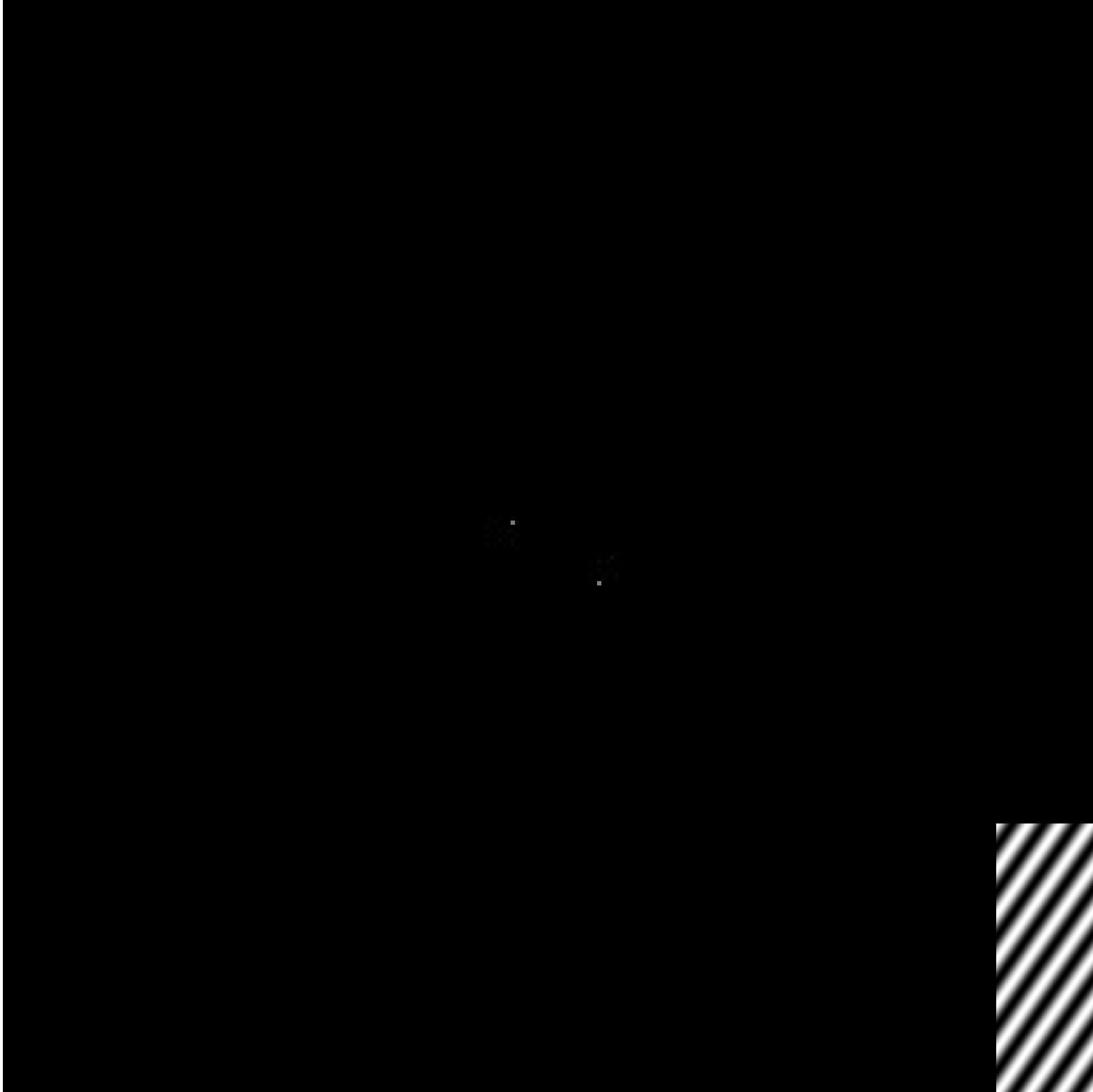
Hoy veremos:

- Aplicaciones DFT y recordatorio de la convolución
- Muestreo (Teorema de Shannon)
- Aliasing

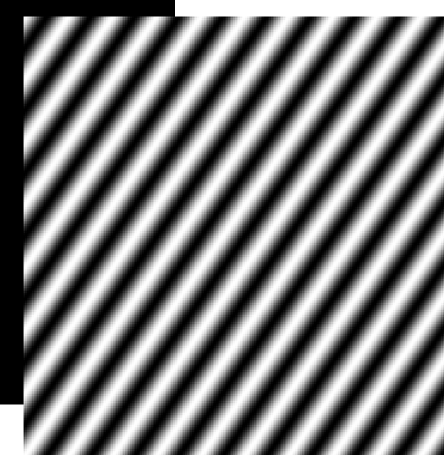
Transformadas de Fourier 2D

Como se ven las DFT de algunas imagenes.
(El modulo)

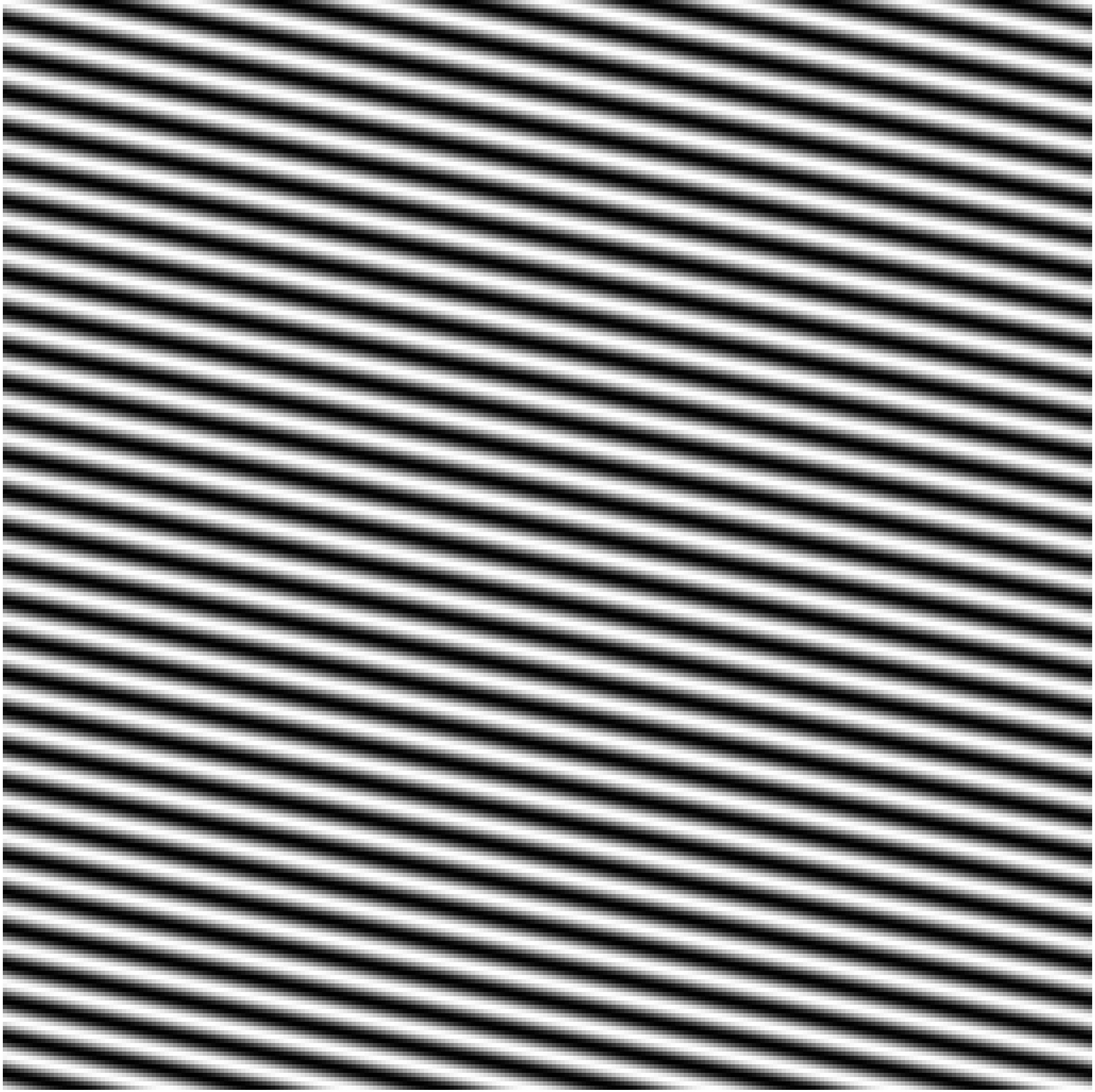
A

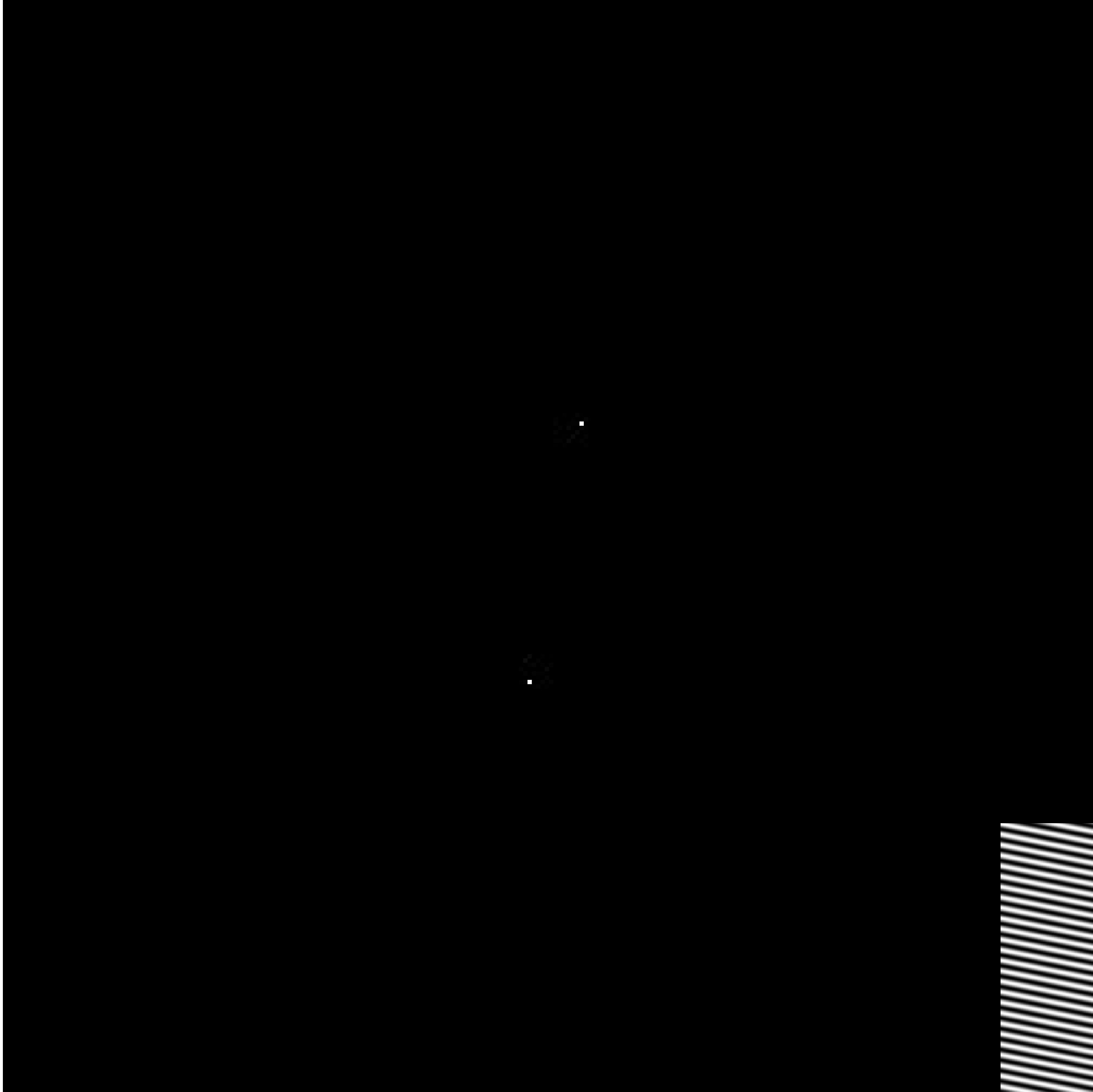


DFT de A

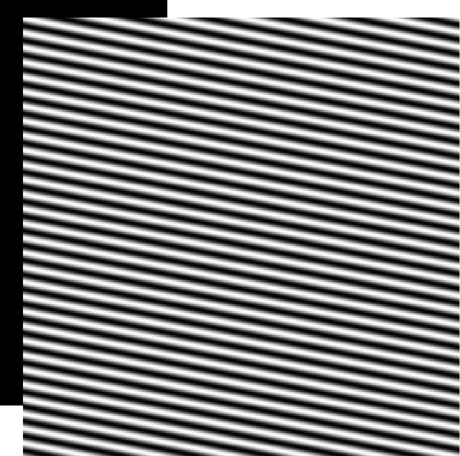


B

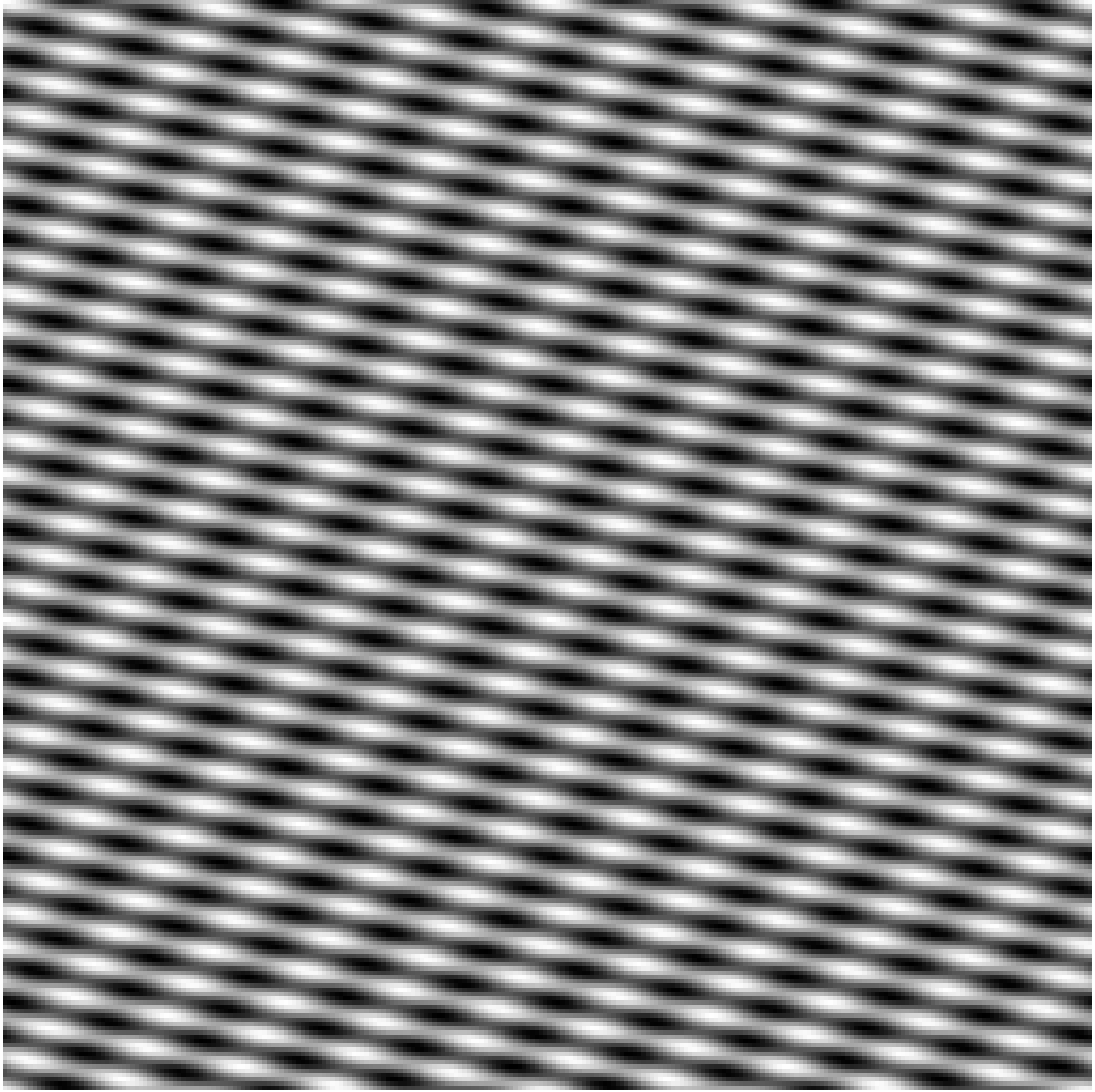


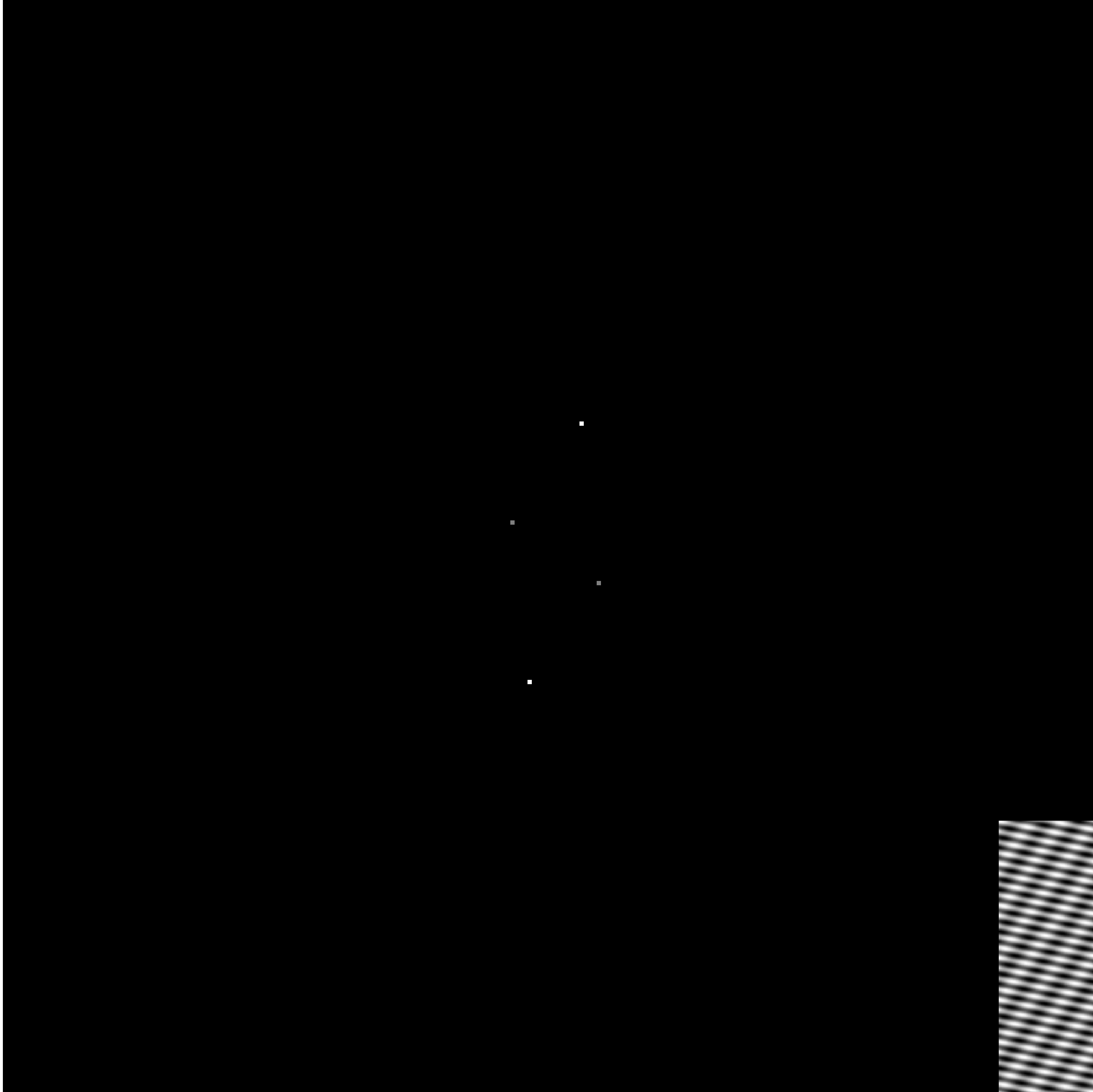


DFT de B

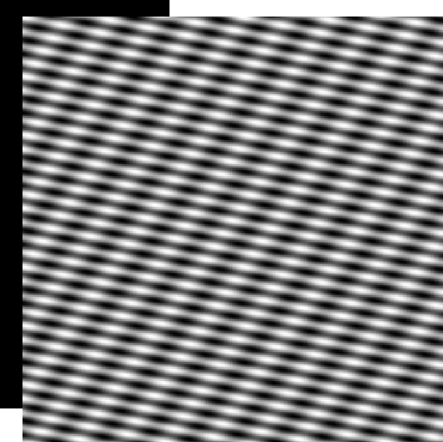


A+B

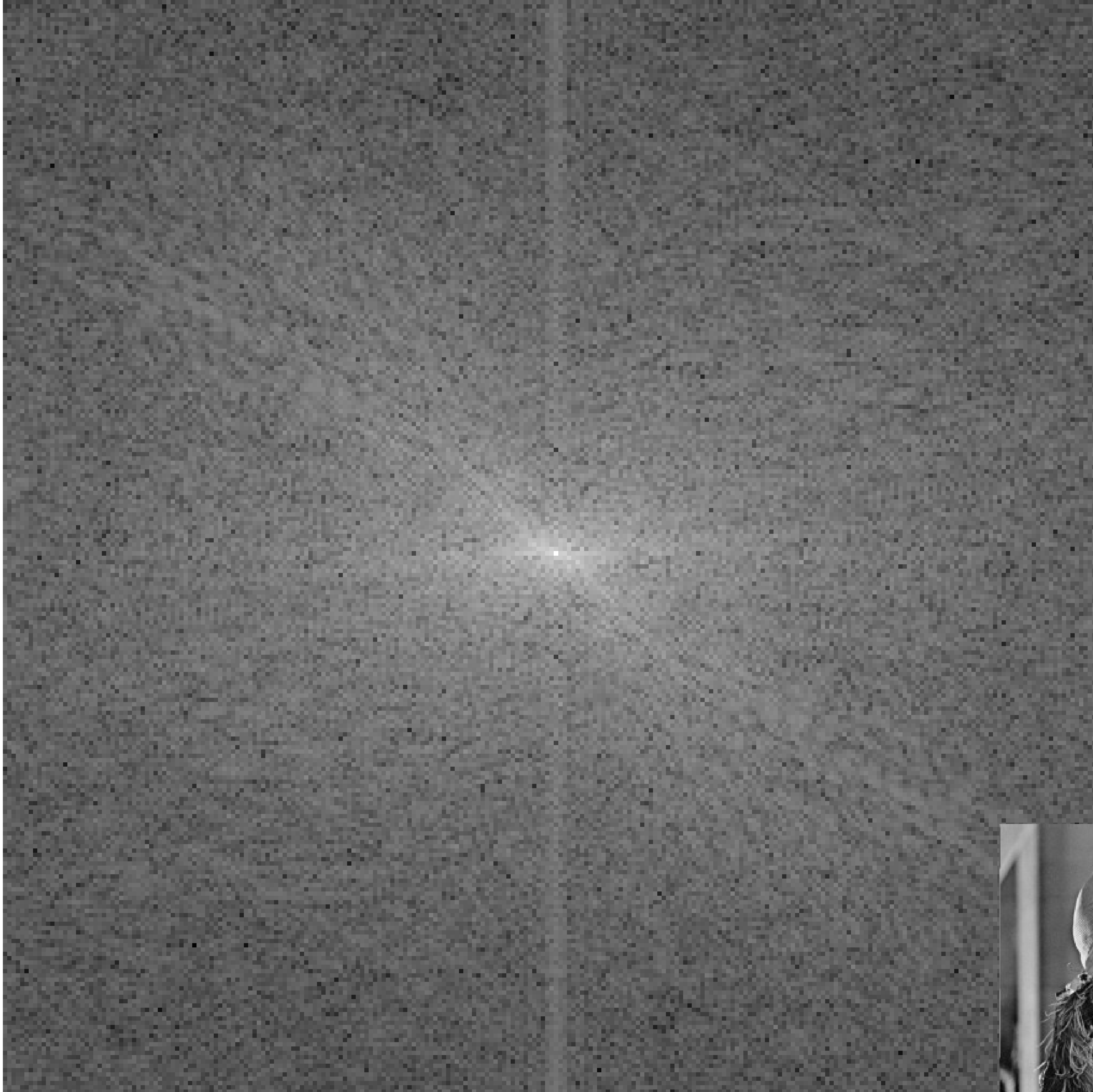




DFT de A+B



Lena



DFT de Lena

Recordar: Interpolación por Fourier

$f : [0, T] \to \mathbb{R}, \quad \text{periódica de period. T}$

Conocemos N muestras: $f_k = f(\frac{kT}{N}), \;\; k = 0...N-1$

Recordar: Interpolación por Fourier

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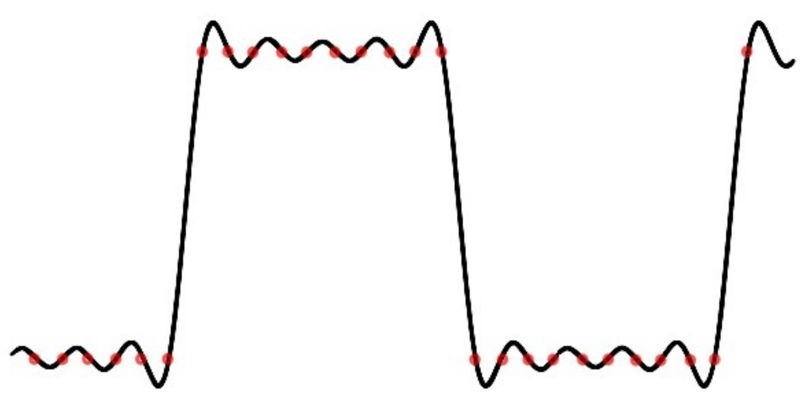
Poly. Trigon. asoc a f_k : $P(t) = \sum_{n=-N/2}^{N/2-1} c_n e^{\frac{2\pi i}{T} nt}$



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Poly. Trigon. asoc a f_k : $P(t) = \displaystyle\sum_{n=-N/2}^{N/2-1} c_n e^{\frac{2\pi i}{T} nt}$

donde: $c_n = \dfrac{1}{N} \displaystyle\sum_{k=0}^{N-1} f_k (w_N)^{nk}, \;\; -\dfrac{N}{2} \leq n \leq \dfrac{N}{2} - 1$

$$w_N = e^{-\frac{2\pi i}{N}}$$

Recordar: Interpolación por Fourier

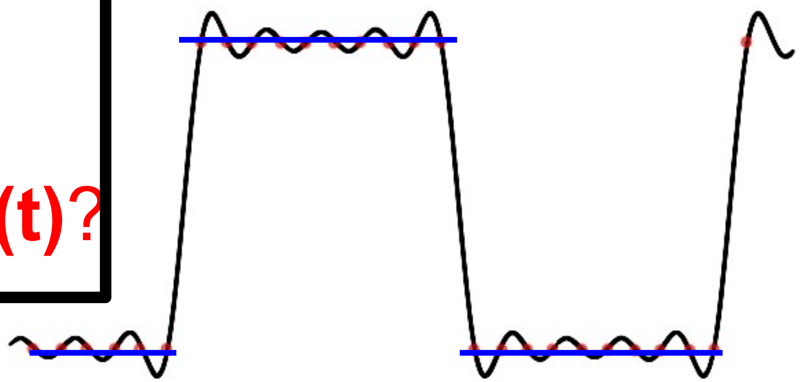
$$f : [0, T] \to \mathbb{R}, \quad \text{periódica de period. T}$$

$$\text{Conocemos N muestras: } f_k = f(\frac{kT}{N}), \;\; k = 0...N-1$$

$$\text{Poly. Trigon. asoc a } f_k : \; P(t) = \sum_{n=-N/2}^{N/2-1} c_n e^{\frac{2\pi i}{T} nt}$$

Pregunta:

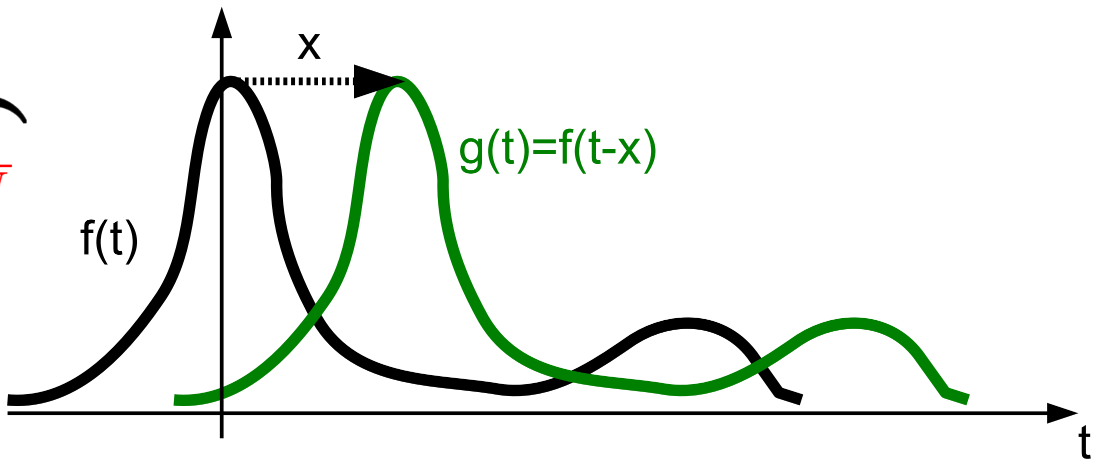
¿Que condiciones imponer sobre **f** y las **muestras** para que **P(t) = f(t)**?



Translación

$$f(t) = \sum_{n=-N/2}^{N/2-1} c_k \overbrace{e^{2\pi i t \frac{k}{N}}}^{w_N^{-kt}}$$

$$g(t) = f(t - x)$$



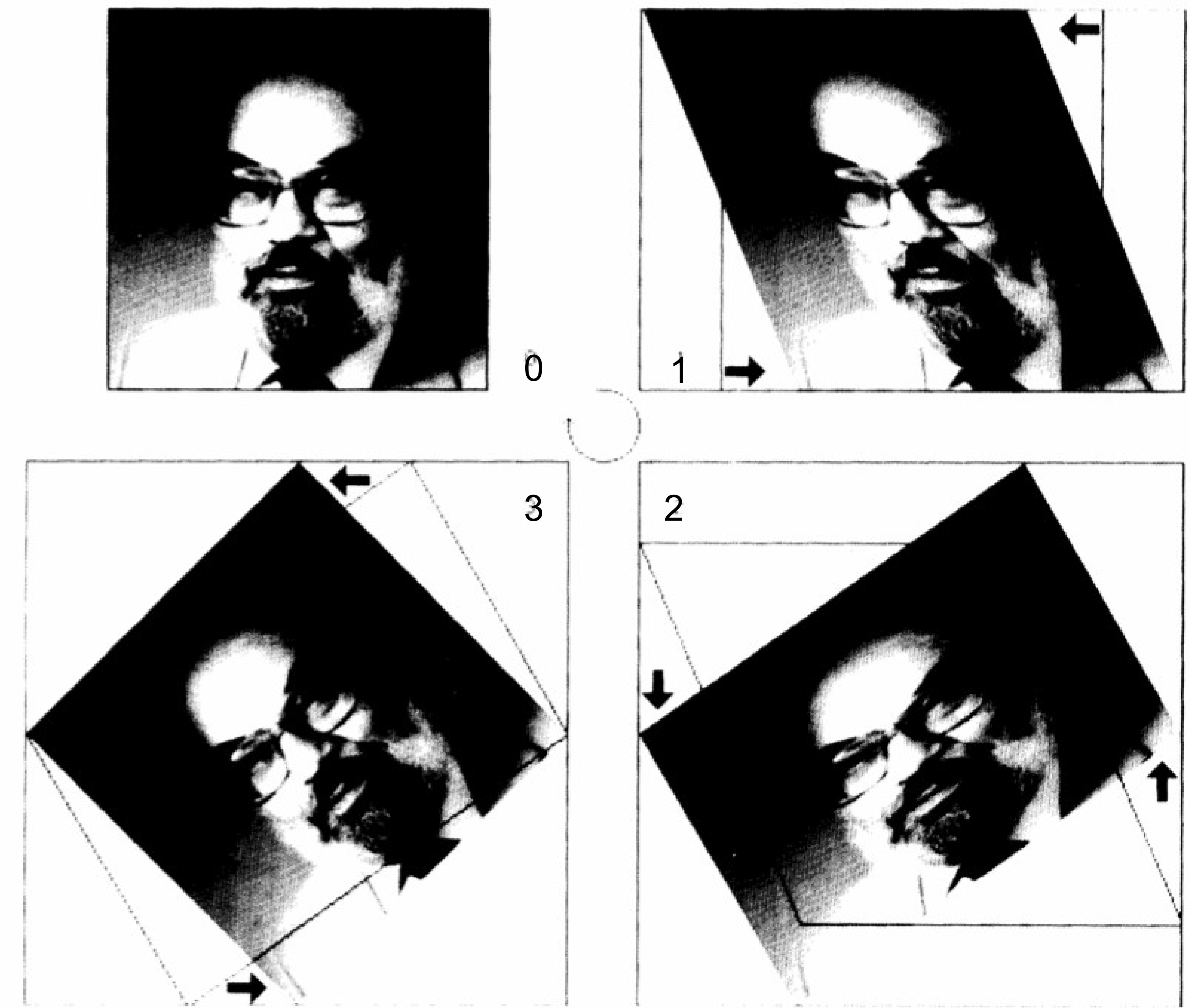
$$f(t-x) = \sum_{n=-N/2}^{N/2-1} c_k e^{2\pi i (t-x) \frac{k}{N}} = \sum_{n=-N/2}^{N/2-1} \underbrace{c_k e^{-2\pi i x \frac{k}{N}}}_{c'_k} e^{2\pi i t \frac{k}{N}}$$

Observar que c'k no depende de t

$$f(t-x) = \sum_{n=-N/2}^{N/2-1} c'_k e^{2\pi i t \frac{k}{N}}$$

Método: $c_k = DFT(f)\ ;\ c'_k = c_k e^{-2\pi i x \frac{k}{N}}\ ;\ g = IDFT(c'_k)$

Rotación como translaciones



Rotación

Resultado al aplicar
50 rotaciones usando
Interpolacion Bilineal

Resultado al aplicar
50 rotaciones usando
Interpolacion por Fourier

Recordar: Convolución discreta

$$f, g : \mathbb{Z} \rightarrow \mathbb{R} \qquad (f * g)[n] = \sum_{i=-\infty}^{\infty} f[i]g[n-i]$$

Si $f = (f_0, ..., f_{N-1}),\ g = (g_0, ..., g_{N-1})$ period. N

Convol. Circular: $$(f * g)[n] = \sum_{i=0}^{N-1} f[i] \underbrace{g[n-i]}_{g[(n-i) \bmod N]}$$

Teorema de la convolucion

$$\mathrm{DFT}(f * g)[k] = N\ \mathrm{DFT}(f)[k]\ \mathrm{DFT}(g)[k] \quad k = 0...N-1$$

$$(f * g)[k] = N\ \mathrm{IDFT}(\mathrm{DFT}(f)[k]\ \mathrm{DFT}(g)[k]) \quad k = 0...N-1$$

Recordar: Convolución discreta

Website:
The Joy of convolution

$$f, g : \mathbb{Z} - \quad [n] = \sum_{i=-\infty}^{\infty} f[i]g[n-i]$$

Si $\quad = (g_0, ..., g_{N-1})$ period. N

$$(f * g)[n] = \sum_{i=0}^{N-1} f[i] \quad \underbrace{g[n-i]}_{g[(n-i) \bmod N]}$$

Teorema de la convolucion

$$\mathrm{DFT}(f * g)[k] = N\ \mathrm{DFT}(f)[k]\ \mathrm{DFT}(g)[k] \quad k = 0 \ldots N-1$$

$$(f * g)[k] = N\ \mathrm{IDFT}(\mathrm{DFT}(f)[k]\ \mathrm{DFT}(g)[k]) \quad k = 0 \ldots N-1$$

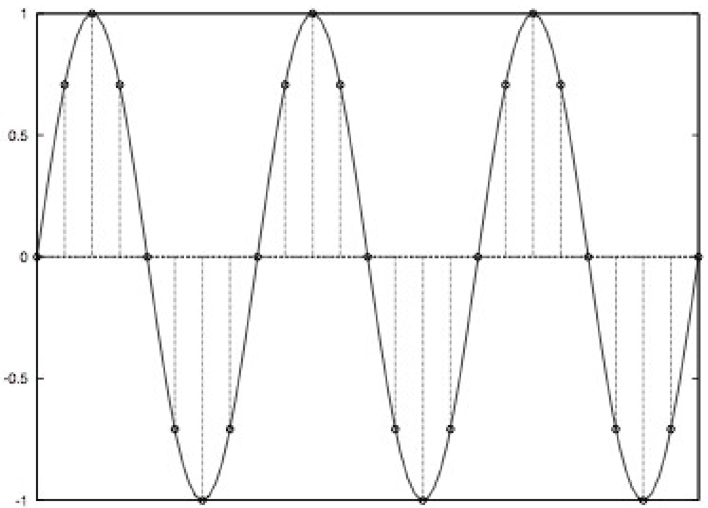
Alias en imagenes

Submuestreos

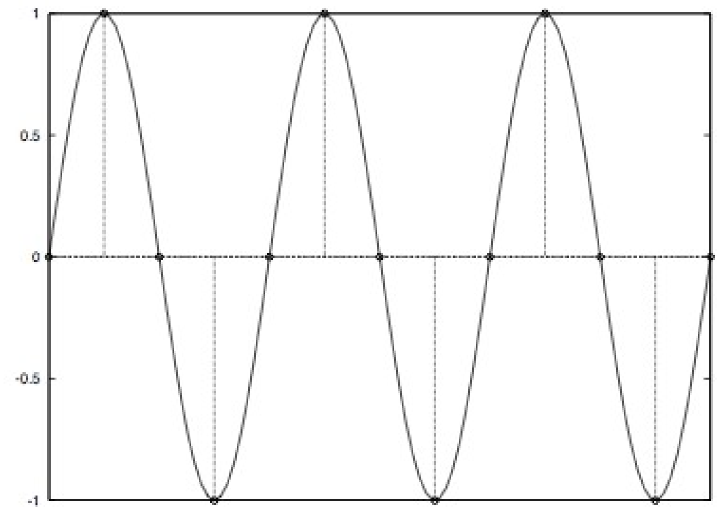
Con alias

Sin alias

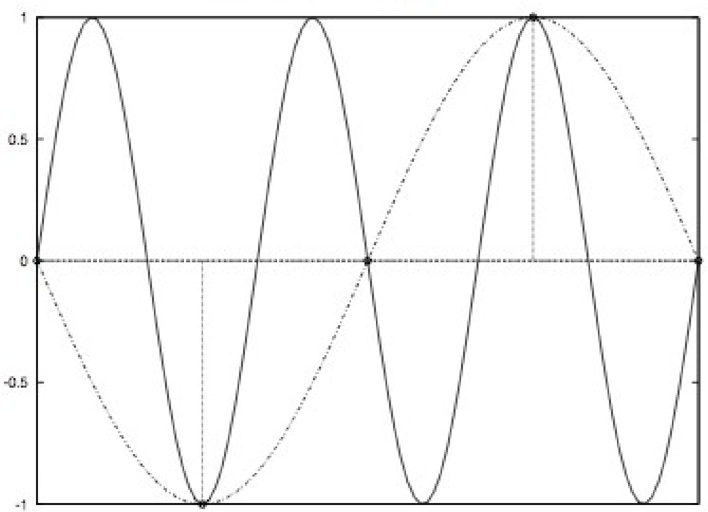
Alias en 1D



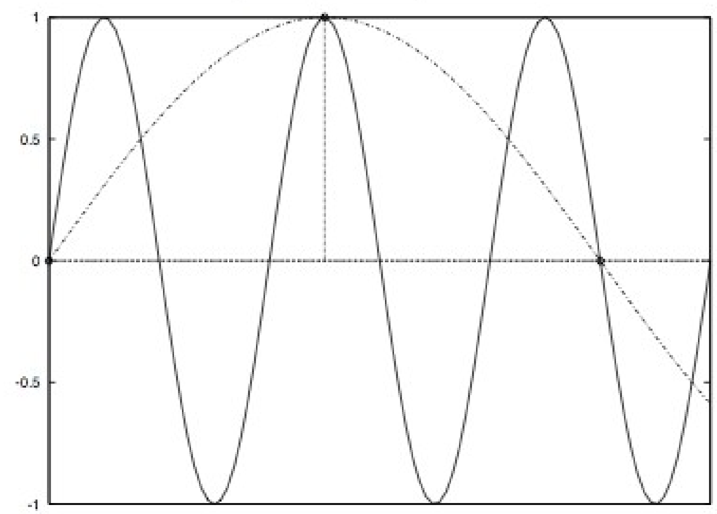
a) $T_m = 1/8\,T$



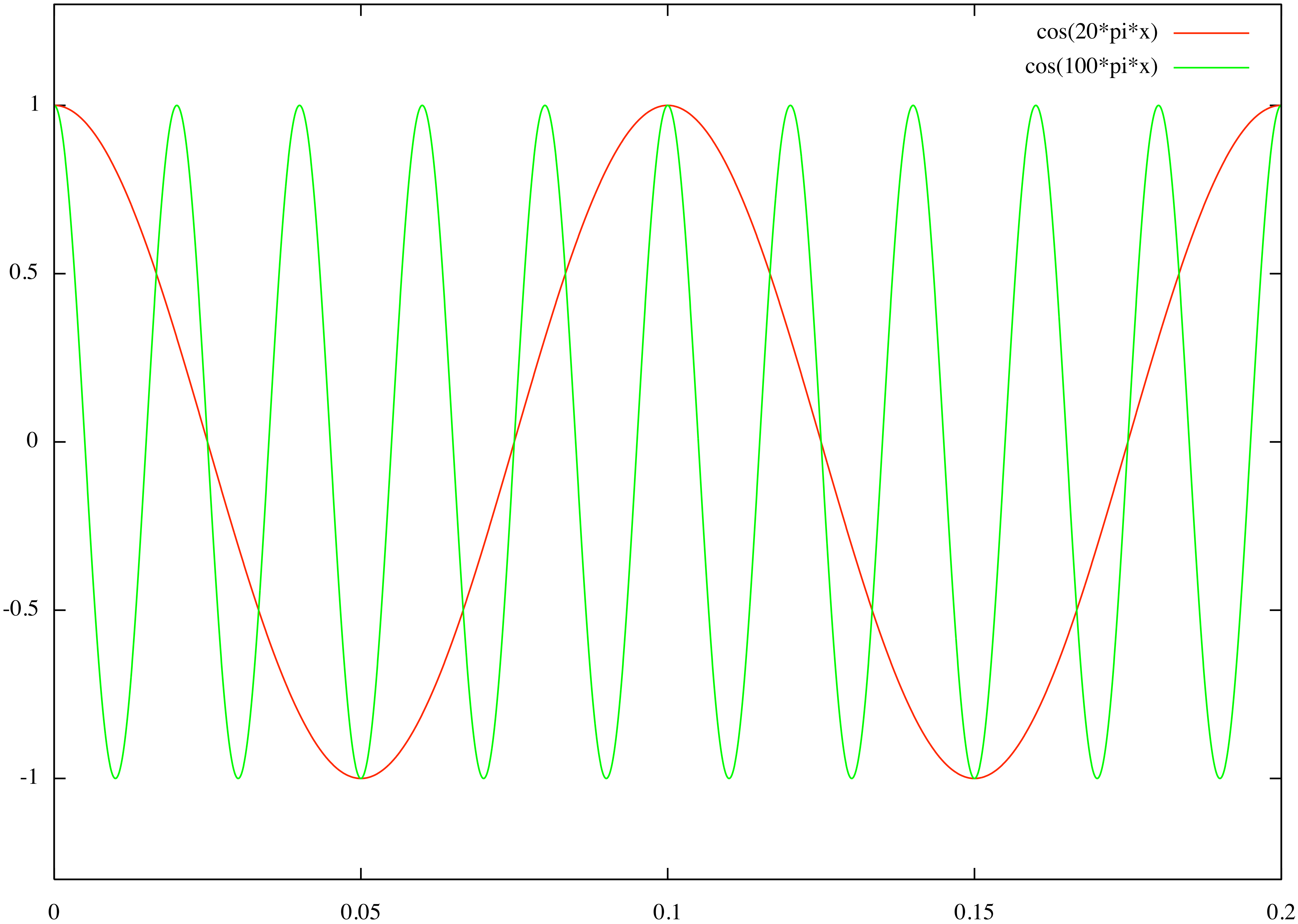
b) $T_m = 1/4\,T$



c) $T_m = 3/4\,T$



d) $T_m = 5/4\,T$


cos(20*pi*x)
cos(100*pi*x)
1
0.5
0
-0.5
-1
0
0.05
0.1
0.15
0.2