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Second Conference on Forward Modelling of Sedimentary Systems

From Desert to Deep Marine Depositional Systems

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A Numerical Implementation of Landscape Evolution Models

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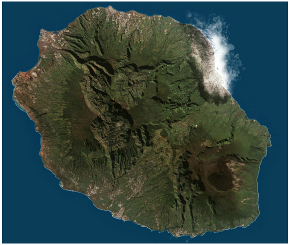
What's Landscape Evolution ?



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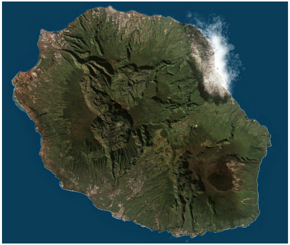
What's Landscape Evolution ?



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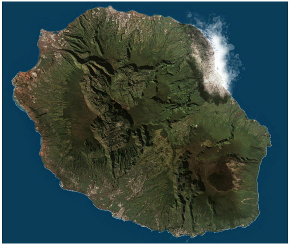
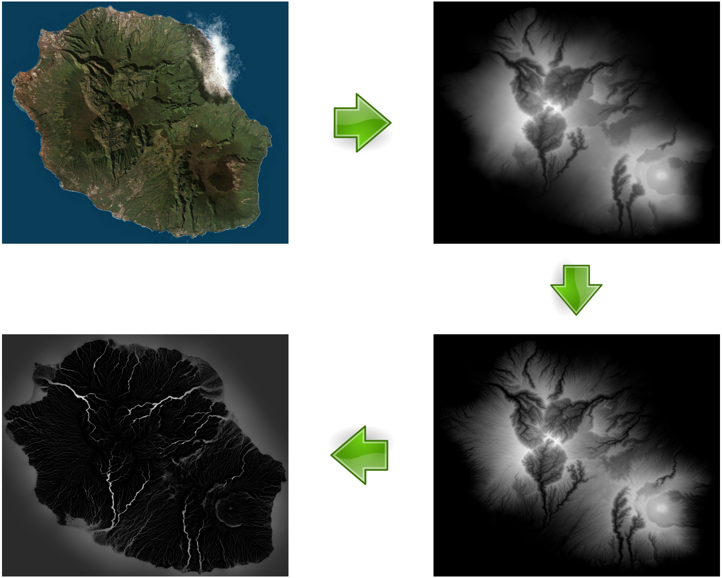
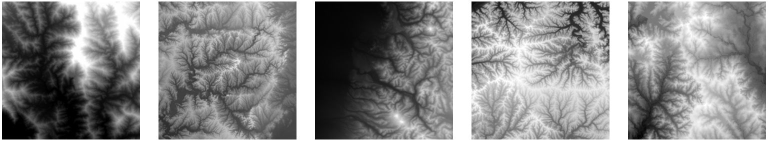


What's Landscape Evolution ?



Motivation

- **Landscape evolution** based on sedimentation, erosion, fluid flow ;
- Also depends on **environmental factors** : soil conditions, human interference, wind, etc.
- Our model is based on previous studies that :
  - rely heavily on **empirical law** rather than physical principles ;
  - contain a **large number** of equations and parameters ;
- Relative **similarity** between landscapes :



Goals :

- Find a **simple** landscape model describing common morphology in nature ;
- Direct **numerical simulations** on **full** landscapes.

Variables and parameters

Variables :

- z = land surface elevation ;
- θ = water height ;
- ρ = sediment density in water ;
- $z + \theta$ = landscape altitude ;
- $\nabla(z + \theta)$ = landscape slope ;

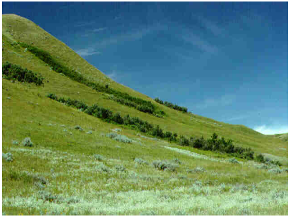
Parameters :

- Uniform rain r at all spatial locations ;
- Sedimentation rate s ;
- Erosion rate e ;
- Creep rate c.

Mass wasting creep

- Creep (c) is the diffusion of soil/rock due to gravitational effects and stress ;
- It depends mainly on **steepness**, with secondary causes in vegetation and soil type ;
- It should be **distinguished from** sedimentation and erosion effects which are based on water transport ;
- Creep causes hills to be **convex upslope** and **concave downslope**
- It tends to smooth the landscape by $c\Delta z$

Creep causes the fence to tilt downslope, as soil shifts downward.



Erosion - stream incision law

- Erosion should be based on stream power at a certain point ;
- Based on amount of water and velocity ;
- Its rate $\frac{\partial z}{\partial t}$ is related to the **slope** of the channel $\nabla(z+\theta)$ and the area of the drainage basin A ;
- Common stream incision law :

$$\boxed{\frac{\partial z}{\partial t} = -eA^m|\nabla(z+\theta)|^n}$$

with $n = 2m$ in many cases.

The Loess Plateau in China has the most erodible soil in the world due to wind and water erosion and man-made factors.

Empirical law, but with intuitive **physical sense** when θ is used in place of drainage area A.

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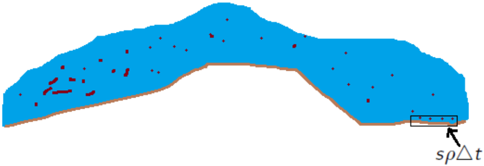
Sedimentation

- Sedimentation occurs as sediments in the water layer settle at the interface between the water and land surface layers ;
- Not based on the total amount of sedimentation in the water, but rather the concentration ;
- Only sediments from the layer of water directly above the land surface layer will settle at the bottom ;
- This gives the sedimentation equation : $\boxed{\frac{\partial z}{\partial t} = s\rho}$



Transport equations

Water level

- The water transport flux is based on the slope of the landscape $\nabla(z+\theta)$ and the amount of water θ present ;
- It follows a **transport equation**

$$\boxed{\frac{\partial \theta}{\partial t} = div\left[\theta\left(\nabla\left(z+\theta\right)\right)\right] + r}$$

Sediment

- The total sediment $\theta\rho$ is a product of the sediment concentration and the water level ;
- It also follows a **transport equation**

$$\underbrace{\frac{\partial(\theta\rho)}{\partial t}}_{\text{total change in sediment}} = \underbrace{div\left[\theta\rho\left(\nabla\left(z+\theta\right)\right)\right]}_{\text{change due to water flow}} - \underbrace{\left(\frac{\partial z}{\partial t} - c\Delta z\right)}_{\text{change from landscape elevation}}$$

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Our proposed model

Three-equations LEM :

- Water conservation and transport law (eq. 1) ;
- Landscape evolution (eq. 2) ;
- Sedimentation mass conservation (eq. 3).

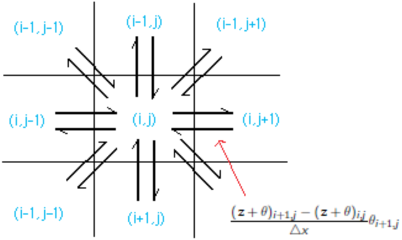
$$\frac{\partial \theta}{\partial t} = div\left[\theta \nabla \left(z + \theta\right)\right] + r \tag{1}$$

$$\frac{\partial z}{\partial t} = c\Delta z - e\theta^m |\nabla \left(z + \theta\right)|^{2m} + s\rho \tag{2}$$

$$\frac{\partial \left(\theta\rho\right)}{\partial t} = \left(c\Delta z - \frac{\partial z}{\partial t}\right) + div\left[\theta\rho\nabla\left(z+\theta\right)\right] \tag{3}$$

Algorithms details

- 8 neighbours ;
- The scheme is **conservative** by construction and **consistent** with the PDE ;



- Both transport equations are approximated by a (half-centred) **finite volume scheme** ;
- The **Water network** must be **initialized** ;
- Impossibility to get an absolute amount of time for a given evolution.

Stopping criteria : *number of iterations or percentage of landscape erosion.*

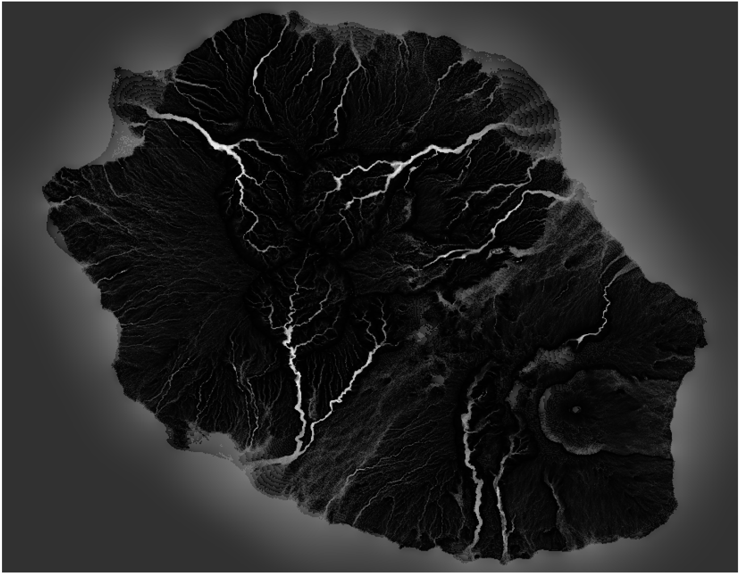
Full evolution - La Réunion

Original landscape - DEM

Full evolution - La Réunion

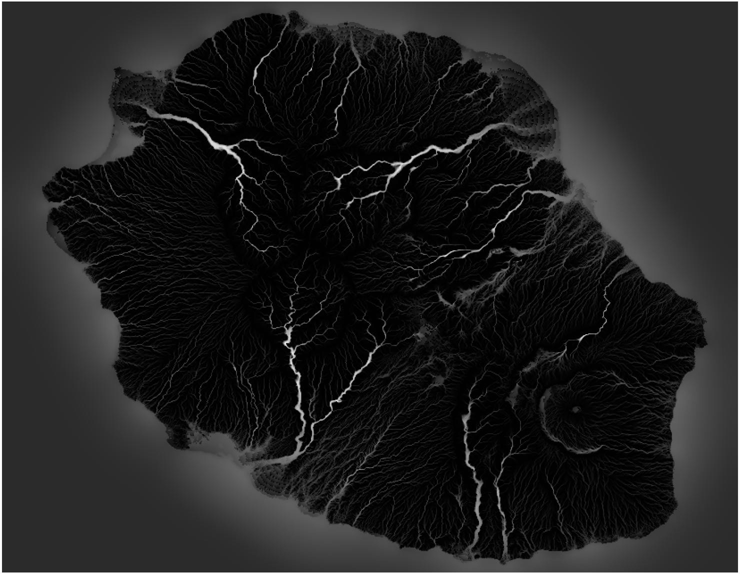
Final landscape (10.5%) - DEM

Full evolution - La Réunion



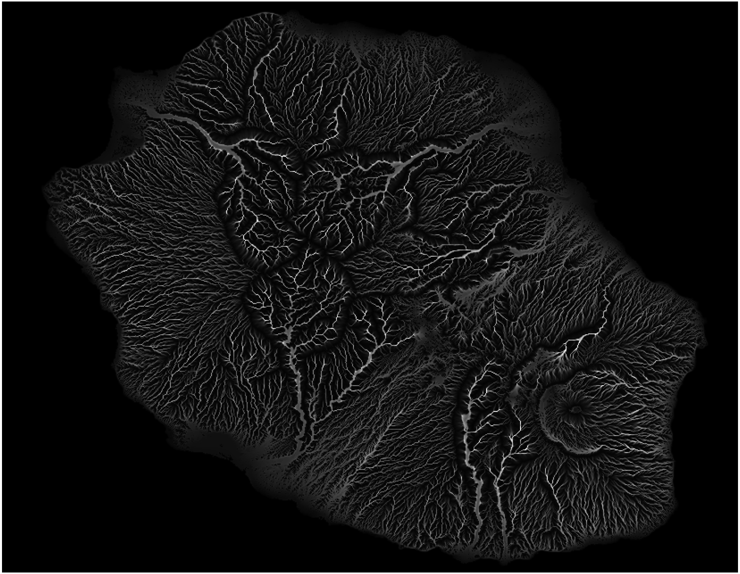
Initial water

Full evolution - La Réunion



Final water

Full evolution - La Réunion



Final sediment

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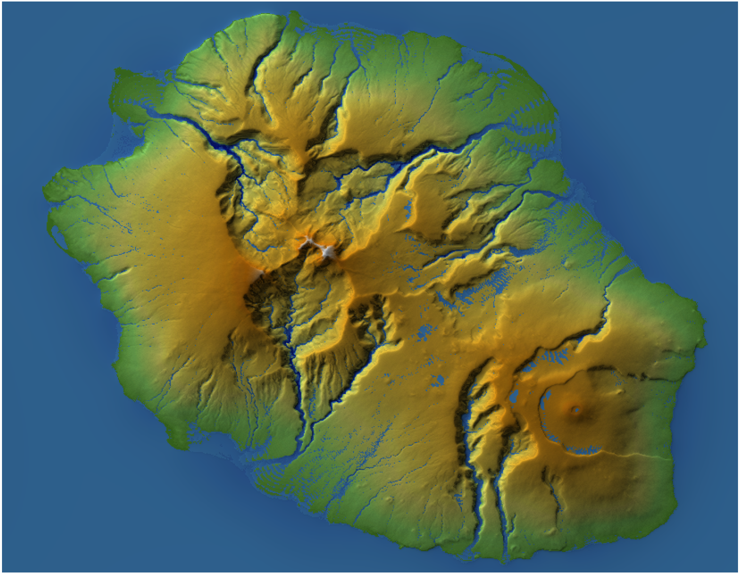
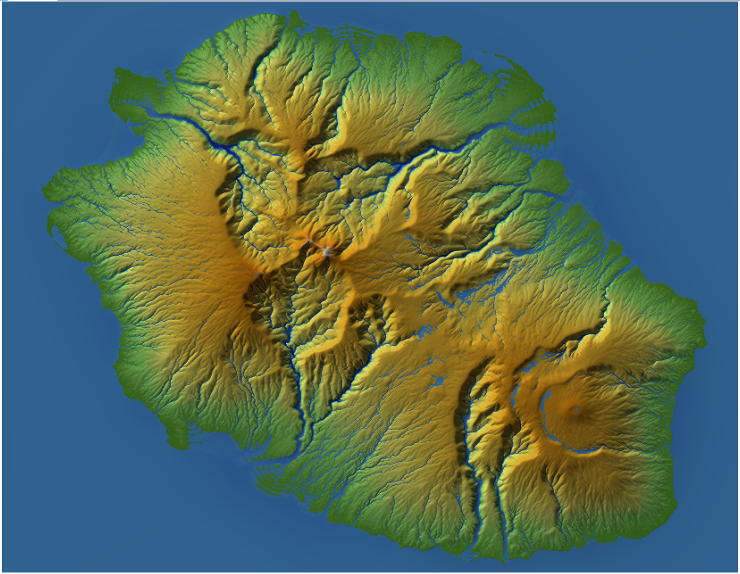
Full evolution - La Réunion



Original landscape - false color

Full evolution - La Réunion

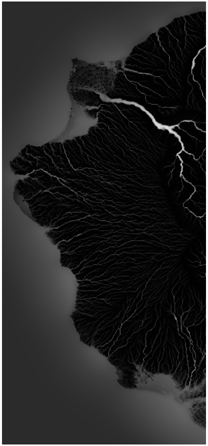


Final landscape (10.5%) - false color

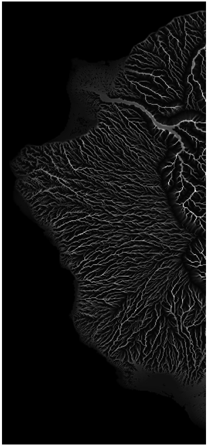
Influence of creep - $c = 10^{-4}$

Final landscape

Final landscape



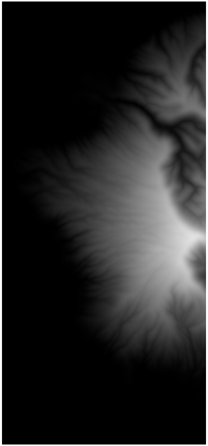
Final water



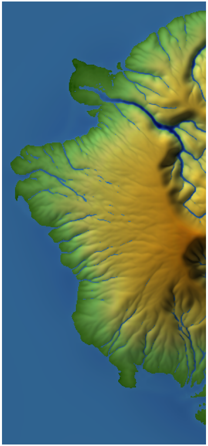
Sediment

10.5% of the landscape have been removed.

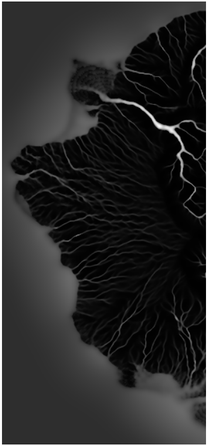
Influence of creep - $c = 5.10^{-3}$



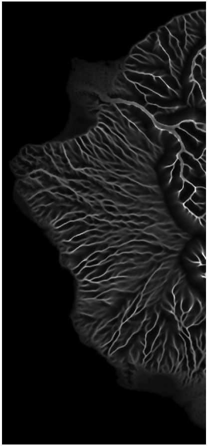
Final landscape



Final landscape



Final water



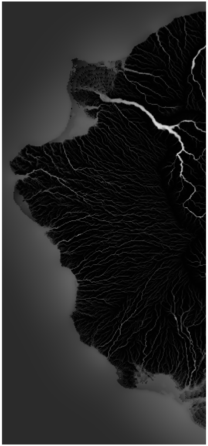
Sediment

11.2% of the landscape have been removed.

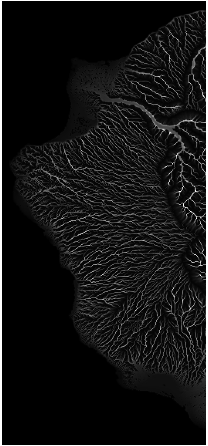
Influence of sedimentation - $s = 10^{-7}$

Final landscape

Final landscape



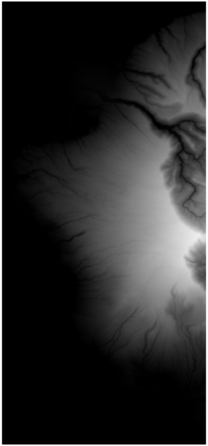
Final water



Sediment

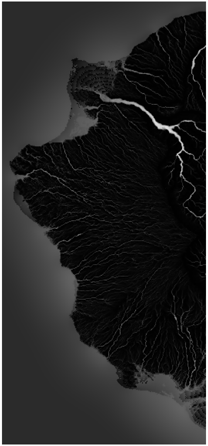
10.5% of the landscape have been removed.

Influence of sedimentation - $s = 10^{-5}$

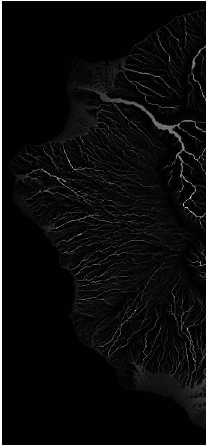


Final landscape

Final landscape



Final water



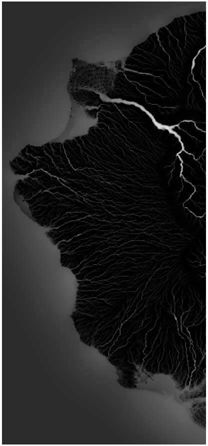
Sediment

1.6% of the landscape have been removed.

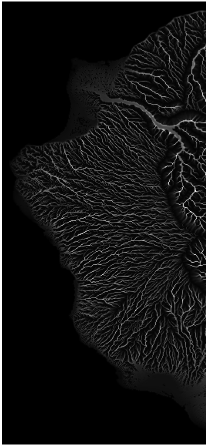
Influence of erosion - $e = 1$

Final landscape

Final landscape



Final water



Sediment

10.5% of the landscape have been removed.

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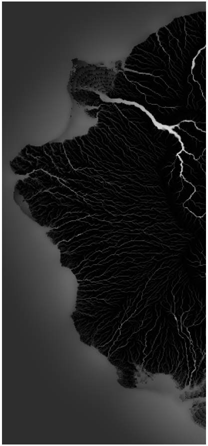
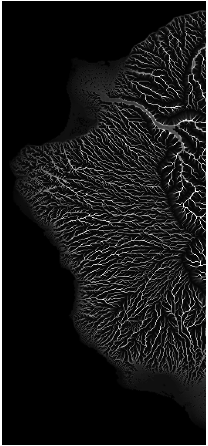
Influence of erosion - $e = 2$

Final landscape

Final landscape



Final water



Sediment

21.5% of the landscape have been removed.

Application to Sharpening River Networks

Problem :

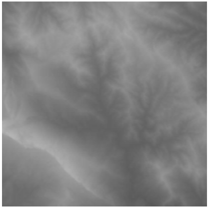
- DEM generally comes with noise/blur/quantification, are imprecise.

Idea :

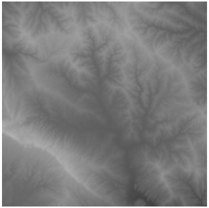
- Apply short time evolution with small erosion and no sedimentation/creep.

Advantage :

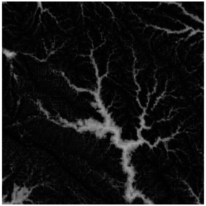
- River networks can be extracted clearly ;
- DEM becomes hydrologically coherent ;
- Gain of networks precision.



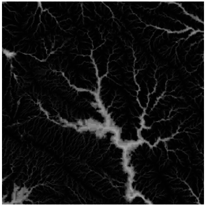
Original landscape



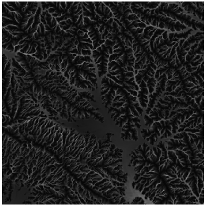
Final landscape



Initial water



Final water



Final sediment

On-line demonstration

Landscape Evolution Model

article | demo | archive

This algorithm models a lanscape evolution. The input image must be in gray level (whiter means higher).

First the landscape will be initialized by a Fast Water Initialization, then the three-equation LEM will be applied.

Select Data

Click on an image to use it as the algorithm input.

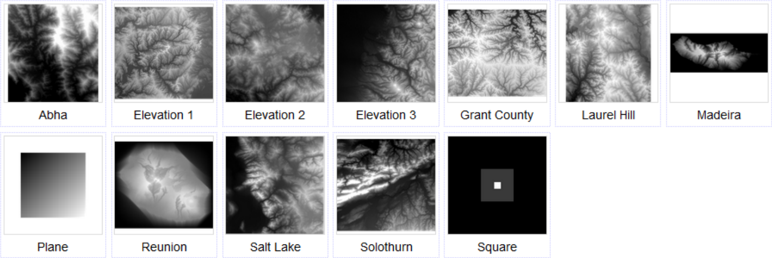


image credits

Upload Data

Upload your own image files to use as the algorithm input.

input image [Parcourir...] Aucun fichier sélectionné. [upload]

http://dev.ipol.im/~ibal/ipol_demo/LEM/ (user : demo, pwd : demo)

Conclusions

- Some popular landscape models :
  - Can describe evolution of **certain** landscapes well ;
  - Often have **too many parameters** for generality ;
  - Based on **empirical** observations.

- Presentation of a landscape evolution model :
  - based on **mathematical conservation laws** ;
  - restricted to **3 equations** with **3 variables**, **4 parameters** ;
  - able to work on full SRTMs within a minute ;
  - verifies several commonly observed landscape features.

Future work :

- Automatic selection of parameters to ensure that for mature landscapes the evolved landscape stays close in character to the original.

Creep evolution

- As for all evolutions, works on **8 neighbours** ;
- Evolved **separately** from erosion and sedimentation for efficiency ;

Input : previous landscape height ${h_f}^{n-1}$.

Output : current landscape height ${h_f}^{n}$.

---

for $(i,j) \in [\![0, NR-1]\!] \times [\![0, NC-1]\!]$

$$\Delta_1 = {h_f}^{n-1}(i,j-1) + {h_f}^{n-1}(i,j+1) + {h_f}^{n-1}(i-1,j) + {h_f}^{n-1}(i+1,j) - 4{h_f}^{n-1}(i,j)$$

$$\Delta_2 = {h_f}^{n-1}(i-1,j-1) + {h_f}^{n-1}(i-1,j+1) + {h_f}^{n-1}(i+1,j-1) + {h_f}^{n-1}(i+1,j+1) - 4{h_f}^{n-1}(i,j)$$

$$ {h_f}^{n}(i,j) = {h_f}^{n-1}(i,j) + \delta_t c \left(\Delta_1 + \frac{1}{\sqrt{2}}\Delta_2\right)$$

endfor

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Water and sedimentation evolution

Input : previous water level θ^{n-1} ;
Input : landscape height h_f ;
Input : previous water times concentration λ^{n-1}.

Output : current water level θ^{n} ;
Output : current water times concentration λ^{n}.

---

for $(i,j) \in [\![0, NR-1]\!] \times [\![0, NC-1]\!]$

$\tau_\theta = \tau_\lambda = 0$

Compute the transfers over the 8 neighbors
for $(i_n, j_n) \in \{(0,\pm 1), (\pm 1, 0), (\pm 1, \pm 1), (\pm 1, \mp 1)\}$

Compute the steep
$steep = \theta^{n-1}(i_n, j_n) + h_f(i_n, j_n) - \theta^{n-1}(i,j) - h_f(i,j)$

Udpate the transfer
if $steep > 0$
$\tau_\theta = \tau_\theta + \omega * \delta_t * steep * \theta^{n-1}(i_n, j_n)$
$\tau_\lambda = \tau_\lambda + \omega * \delta_t * steep * \lambda^{n-1}(i_n, j_n)$
else
$\tau_\theta = \tau_\theta + \omega * \delta_t * steep * \theta^{n-1}(i,j)$
$\tau_\lambda = \tau_\lambda + \omega * \delta_t * steep * \lambda^{n-1}(i,j)$
endif
endfor

Update the water and water times concentration level
$\theta^{n}(i,j) = \theta^{n-1}(i,j) + \tau_\theta + \delta_t * r \qquad \lambda^{n}(i,j) = \lambda^{n-1}(i,j) + \tau_\lambda$
endfor

Where ω is a ponderation : $\omega = 1$ if $(i_n, j_n) = (0, \pm 1)$ or $(\pm 1, 0)$ and $\omega = \frac{1}{\sqrt{2}}$ otherwise.

Algorithm - landscape evolution

Input : previous landscape height ${h_f}^{n-1}$;
Input : current water level θ ;
Input : previous water times concentration λ^{n-1}.

Output : current landscape height ${h_f}^{n}$;
Output : current water times concentration λ^{n}.

for $(i,j) \in [\![0, NR-1]\!] \times [\![0, NC-1]\!]$

$\delta_e = 0$

Compute the erosion over the 8 neighbors

$h_{\pm 1} = {h_f}^{n-1}(i,j) + \theta(i,j) - {h_f}^{n-1}(i, j\pm 1) - \theta(i, j\pm 1)$

$v_{\pm 1} = {h_f}^{n-1}(i,j) + \theta(i,j) - {h_f}^{n-1}(i\pm 1, j) - \theta(i\pm 1, j)$

$\delta_e = \delta_e + (\max\{\max\{h_{+1}, h_{-1}\}, 0\})^2 + (\max\{\max\{v_{+1}, v_{-1}\}, 0\})^2$

$l_{\pm 1} = {h_f}^{n-1}(i,j) + \theta(i,j) - {h_f}^{n-1}(i\pm 1, j\pm 1) - \theta(i\pm 1, j\pm 1)$

$r_{\pm 1} = {h_f}^{n-1}(i,j) + \theta(i,j) - {h_f}^{n-1}(i\pm 1, j\mp 1) - \theta(i\pm 1, j\mp 1)$

$\delta_e = \delta_e + \frac{1}{2}(\max\{\max\{l_{+1}, l_{-1}\}, 0\})^2 + \frac{1}{2}(\max\{\max\{r_{+1}, r_{-1}\}, 0\})^2$

Get the final amount to erode

$\delta_e = -e_e\,(\theta(i,j))^m\,(\delta_e)^m$

Compute the sedimentation

$\delta_s = e_s \frac{\lambda^{n-1}(i,j)}{\theta(i,j)}$

Landscape to erode

$\delta = \delta_t(\delta_e + \delta_s)$

Update the landscape height

${h_f}^{n}(i,j) = {h_f}^{n-1}(i,j) + \delta$

Update the water times concentration

$\lambda^{n}(i,j) = \lambda^{n-1}(i,j) - \delta$

endfor