Image texture synthesis using spot noise and phase randomization

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Outline

Outline

What is a texture?

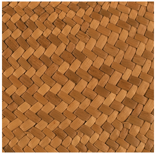
A minimal definition of a **texture** image is an "image containing repeated patterns" [Wei et al., 2009].
The family of patterns reflects a certain amount of randomness, depending on the nature of the texture.
Two main subclasses:

- The ***micro-textures***.



- The ***macro-textures***, constitued of small but discernible objects.





Textures and scale of observation

Depending on the **viewing distance**, the same objects can be perceived either as

- a micro-texture,
- a macro-texture,
- a collection of individual objects.

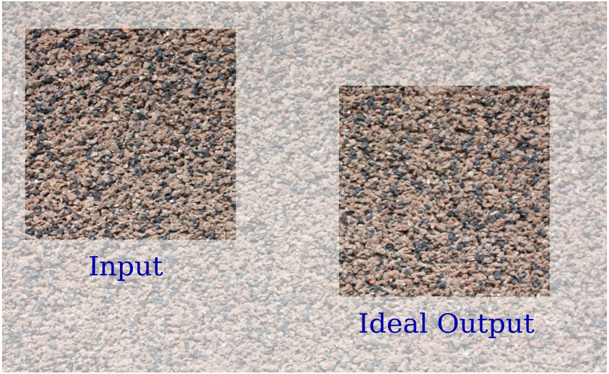
Micro-texture

Macro-texture

Some pebbles

Texture synthesis

Texture Synthesis: Given an input texture image, produce an output texture image being both visually similar to and pixel-wise different from the input texture.



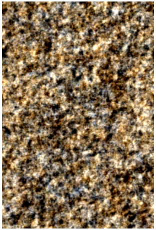
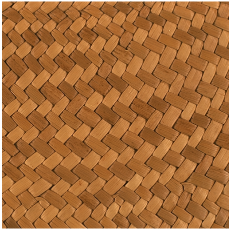
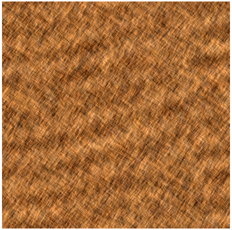
The output image should ideally be perceived as another part of the same large piece of homogeneous material the input texture is taken from.

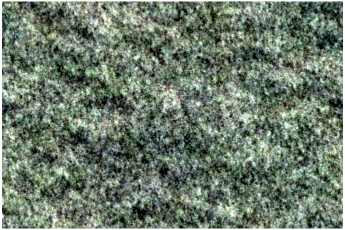
Texture synthesis by phase randomization

- Sucessful examples with micro-textures:

Texture synthesis by phase randomization

- Failure examples with macro-textures:

Outline

Framework

- We work with discrete digital images $u \in \mathbb{R}^{M \times N}$ indexed on the set $\Omega = \{0, \dots, M-1\} \times \{0, \dots, N-1\}$.
- Each image is extended by periodicity:

$$u(k,l) = u(k \mod M, l \mod N) \quad \text{for all } (k,l) \in \mathbb{Z}^2.$$

- Consequence: Translation of an image:

Discrete Fourier transform of digital images

Definition:

- The **discrete Fourier transform (DFT)** of u is the **complex-valued** image $\hat{u}$ defined by:

$$\hat{u}(s,t) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} u(k,l) e^{-\frac{2iks\pi}{M}} e^{-\frac{2ilt\pi}{N}}, \quad (s,t) \in \Omega.$$

- $|\hat{u}|$: **Fourier modulus** of u.
- $\arg(\hat{u})$: **Fourier phase** of u.

Symmetry property:

- Since u is real-valued, $\hat{u}(-s,-t) = \overline{\hat{u}(s,t)}$.
  $\Rightarrow$ the modulus $|\hat{u}|$ is even and the phase $\arg(\hat{u})$ is odd.

Computation:

- The Fast Fourier Transform algorithm computes $\hat{u}$ in $\mathcal{O}(MN\log(MN))$ operations.
- Efficient FFT implementation: **FFTW** library, a **C/C++** library (used in Matlab).

FFTW = **F**astest **F**ourier **T**ransform in the **W**est

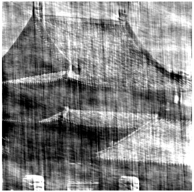
Modulus and phase of a digital image

Exchanging the modulus and the phase of two images:
[Oppenheim and Lim, 1981]

Image 1

Image 2

Modulus of 1
& phase of 2



Modulus of 2
& phase of 1

- Geometric contours are mostly contained in the phase.

Modulus and phase of a digital image

Exchanging the modulus and the phase of two images: [Oppenheim and Lim, 1981]

Image 1

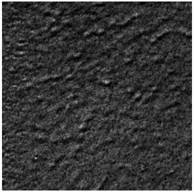
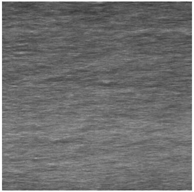


Image 2

Modulus of 1
& phase of 2



Modulus of 2
& phase of 1

- Textures are mostly contained in the modulus.

Modulus and phase of a digital image

Exchanging the modulus and the phase of two images: [Oppenheim and Lim, 1981]

Image 1

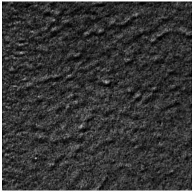
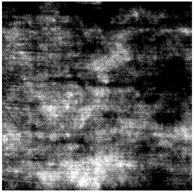
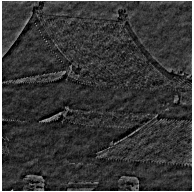


Image 2

Modulus of 1
& phase of 2





Modulus of 2
& phase of 1

- Geometric contours are mostly contained in the phase.
- Textures are mostly contained in the modulus.

Outline

Random phase textures

- First Julesz theory: The Fourier modulus is a key feature for the perception of a texture.

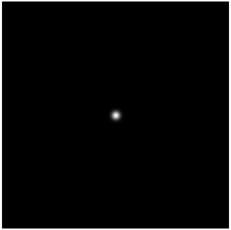
- We call *random phase texture* any image that is perceptually invariant to phase randomization.
- Phase randomization = replace the Fourier phase by a random phase.
- Definition: A random image $\theta \in \mathbb{R}^{M \times N}$ is a ***random phase*** if
  1. Symmetry: θ is odd:
  $$\forall (s,t) \in \Omega, \theta(-s,-t) = -\theta(s,t).$$
  2. Distribution: Each component $\theta(s,t)$ is
     - uniform over the interval $]-\pi,\pi]$ if $(s,t) \notin \left\{(0,0), \left(\frac{M}{2},0\right), \left(0,\frac{N}{2}\right), \left(\frac{M}{2},\frac{N}{2}\right)\right\}$,
     - uniform over the set $\{0,\pi\}$ otherwise.
  3. Independence: For each subset $\mathcal{S} \subset \Omega$ that does not contain distinct symmetric points, the r.v. $\{\theta(s,t) | (s,t) \in \mathcal{S}\}$ are independent.

- *Random phase textures* constitute a “limited” subclasse of the set of textures.

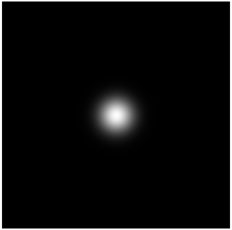
Random Phase Noise (RPN)

- Texture synthesis algorithm: ***random phase noise*** **(RPN)**: [van Wijk, 1991]

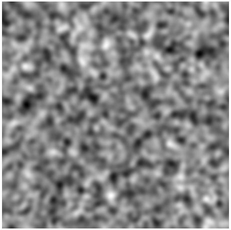
1. Compute the DFT $\hat{h}$ of the input h.
2. Compute a random phase θ using a pseudo-random number generator.
3. Set $\hat{Z} = \left|\hat{h}\right| e^{i\theta}$ (or $\hat{Z} = \hat{h}e^{i\theta}$).
4. Return Z the inverse DFT of $\hat{Z}$.



Original image h



Modulus $\left|\hat{h}\right|$



RPN associated with h

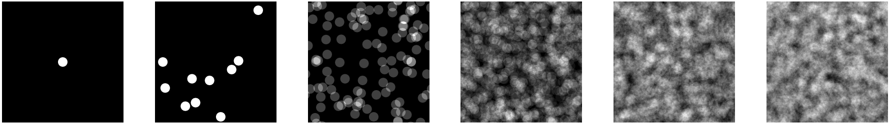
Outline

Discrete spot noise [van Wijk, 1991]

Texture model with two main characteristics: linear superimposition and invariance by translation of the objects.

- Let h be a discrete image called *spot*.
- Let (X_k) be a sequence of i.i.d. r.v. uniformly distributed over $\Omega = \{0, \dots, M-1\} \times \{0, \dots, N-1\}$.
- The **discrete spot noise of order** n **associated with** h is the random image

$$f_n(x) = \sum_{k=1}^{n} h(x - X_k).$$



Spot h | $n = 10$ | $n = 10^2$ | $n = 10^3$ | $n = 10^4$ | $n = 10^5$

Asymptotic discrete spot noise (*ADSN*)

- For texture synthesis we are more particularly interested in the limit of the *DSN*: the ***asymptotic discrete spot noise* (*ADSN*)**.

Expectation and covariance of the random translations:

- $\mathbb{E}(h(x - X_1)) = m$, where m is the arithmetic mean of h.
- $\mathrm{Cov}(h(x - X_1), h(y - X_1)) = C_h(x - y)$ where C_h is the autocorrelation of h:

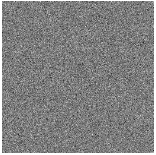
$$C_h(x, y) = \frac{1}{MN} \sum_{t \in \Omega} (h(x - t) - m)(h(y - t) - m), \quad (x, y) \in \Omega.$$

- The *DSN* of order n is the sum of the n i.i.d. random images $h(. - X_k)$.
- Central limit theorem: $\dfrac{f_n - m}{\sqrt{n}}$ converges towards the Gaussian random vector with covariance C_h.

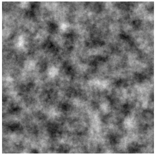
Definition of *ADSN*: the *ADSN* associated with h is the Gaussian vector $\mathcal{N}(0, C_h)$.

Simulation of the ADSN

Definition of *ADSN*: the *ADSN* associated with h is the Gaussian vector $\mathcal{N}(0, C_h)$.



Gaussian white noise: pixels are independent and have Gaussian distribution

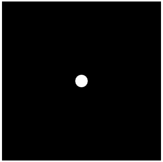


Gaussian vector: pixels have Gaussian distribution and are correlated

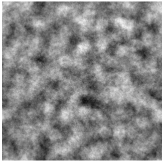
Convolution product: $(f * g)(x) = \sum_{y \in \Omega} f(x-y) g(y), \; x \in \Omega.$

Simulation of the *ADSN*:

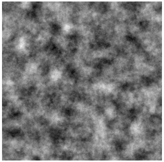
- Let $h \in \mathbb{R}^{M \times N}$ be a an image, m be the mean of h and X be a Gaussian white noise image.
- The random image $\frac{1}{\sqrt{MN}}(h - m) * X$ is the *ADSN* associated with h.



Spot h



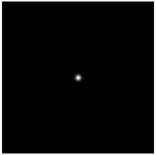
DSN, $n = 10^5$



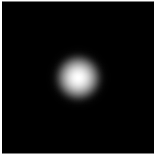
ADSN

Differences between *RPN* and *ADSN*

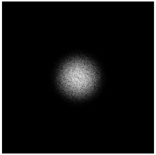
- *RPN* and *ADSN* both have a random phase.
- The Fourier modulus of *RPN* is the one of h.
- The Fourier modulus of *ADSN* is the pointwise multiplication between $\left|\hat{h}\right|$ and a Rayleigh noise.



Spot h

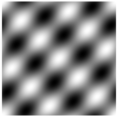


RPN Modulus

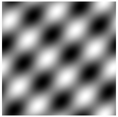


ADSN Modulus

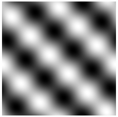
- ***RPN* and *ADSN* are two different processes.**



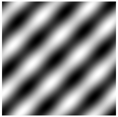
Spot h



RPN



An *ADSN* realization



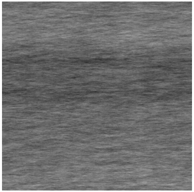
Another *ADSN* realization

Outline

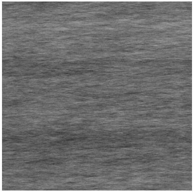
RPN and *ADSN* associated to texture images

- Some textures are relatively well reproduced by *RPN* and *ADSN*.

Original image



RPN



ADSN

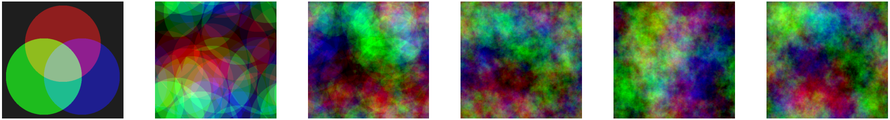
- ... But several developments are necessary to derive texture synthesis algorithms from sample.

Extension to color images

- We use the RGB color representation for color images.
- **Color *ADSN*:** The definition of Discrete Spot Noise extends to color images $h = (h_r, h_g, h_b)$.
- The color *ADSN* Y is the limit Gaussian process obtained in letting the number of spots tends to $+\infty$. It is simulated by:

$$Y = \frac{1}{\sqrt{MN}} \begin{pmatrix} (h_r - m_r \mathbf{1}) * X \\ (h_g - m_g \mathbf{1}) * X \\ (h_b - m_b \mathbf{1}) * X \end{pmatrix}, \quad X \text{ a Gaussian white noise.}$$

- One convolves each color channel with the **same** Gaussian white noise X.



Spot h | $n = 10$ | $n = 10^2$ | $n = 10^3$ | $n = 10^4$ | color *ADSN*

- Phase of color *ADSN*: The same random phase is added to the Fourier transform of each color channel.

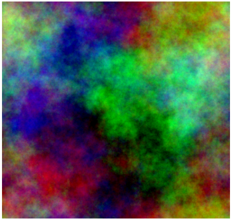
Extension to color images

- **Color *RPN*:** By analogy, the *RPN* associated with a color image $h = (h_r, h_g, h_b)$ is the color image obtained in **adding the same random phase** to the Fourier transform of each color channel.

Original image h

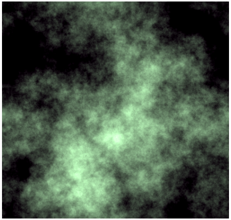
$$\hat{h} = \begin{pmatrix} |\hat{h}_R| e^{i\varphi_R} \\ |\hat{h}_G| e^{i\varphi_G} \\ |\hat{h}_B| e^{i\varphi_B} \end{pmatrix}$$

Color *RPN*



$$\hat{Z} = \begin{pmatrix} |\hat{h}_R| e^{i(\varphi_R+\theta)} \\ |\hat{h}_G| e^{i(\varphi_G+\theta)} \\ |\hat{h}_B| e^{i(\varphi_B+\theta)} \end{pmatrix}$$

"Wrong *RPN*": each channel has the same random phase

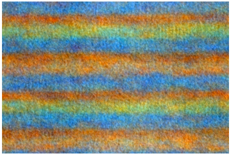


$$\hat{Z}_W = \begin{pmatrix} |\hat{h}_R| e^{i\theta} \\ |\hat{h}_G| e^{i\theta} \\ |\hat{h}_B| e^{i\theta} \end{pmatrix}$$

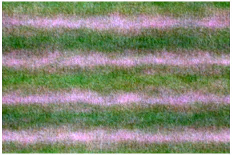
Extension to color images

- Another example with a real-world texture.

Original image *h*



Color *RPN*



"Wrong *RPN*"

- Preserving the original phase displacement between the color channels is essential for color consistancy.
- ...however for most monochromatic textures, there is no huge difference.

Original image *h*

Color *RPN*

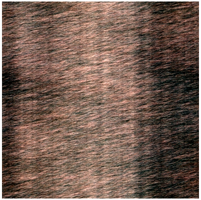
"Wrong *RPN*"

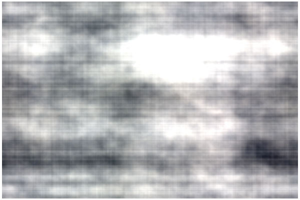
Avoiding artifacts due to non periodicity

- Both *ADSN* and *RPN* algorithms are based on the fast Fourier transform (FFT).
  $\Longrightarrow$ implicit hypothesis of periodicity
- Using non periodic samples yields important artifacts.

Spot h

ADSN





Avoiding artifacts due to non periodicity

- Our solution: Force the periodicity of the input sample.
- The original image h is replaced by its **periodic component** $p = \text{per}(h)$, see L. Moisan's course [Moisan, 2009].
- Definition of the periodic component p of h: p unique solution of

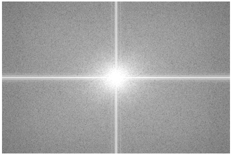
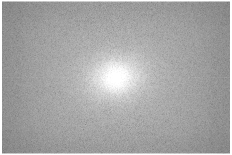
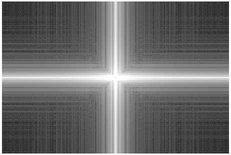
$$\begin{cases} \Delta p = \Delta_i h \\ \text{mean}(p) = \text{mean}(h) \end{cases}$$

where,

$$\Delta f(x) = 4f(x) - \sum_{y \in N_x} f(y) \quad \text{and} \quad \Delta_i f(x) = |N_x \cap \Omega|\, f(x) - \sum_{y \in N_x \cap \Omega} f(y).$$

- p is “visually close” to h (same Laplacian).
- p is fastly computed using the FFT (see the course on retinex for details).

Periodic component: effects on the Fourier modulus

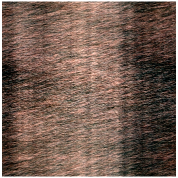
	Image h	Periodic component $p = \text{per}(h)$	Smooth component $s + m$
Images			
Fourier modulus			

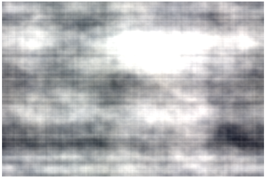
- The application per : $h \mapsto p$ filters out the "cross structure" of the spectrum.

Avoiding artifacts due to non periodicity

Spot h

$ADSN(h)$





$ADSN(p)$

Synthesizing textures having arbitrary large size

To synthesize a texture larger than the original spot h, one computes an "equivalent spot" $\tilde{h}$:

- Copy $p = \text{per}(h)$ in the center of a constant image equal to the mean of h.
- Normalize the variance.
- Attenuate the transition at the inner border.

Spot h — Equivalent spot $\tilde{h}$ — $RPN(h)$ — $RPN\left(\tilde{h}\right)$

Properties of the resulting algorithms

- Both algorithms are fast, with the complexity of the fast Fourier transform [$\mathcal{O}\left(MN \log\left(MN\right)\right)$].
- Visual stability: All the realizations obtained from the same input image are visually similar.

Spot h

$RPN\,1$

$RPN\,2$

$RPN\,3$

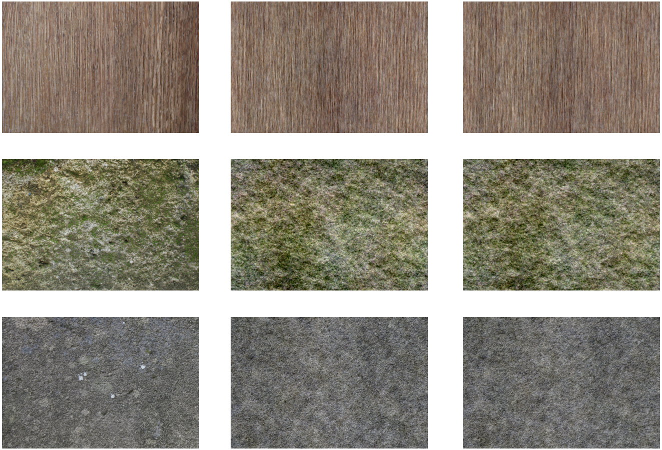
- [ON LINE DEMO]

Outline

Numerical results: similarity of the textures

- In order to compare both algorithms, the same random phase is used for *ADSN* and *RPN*.

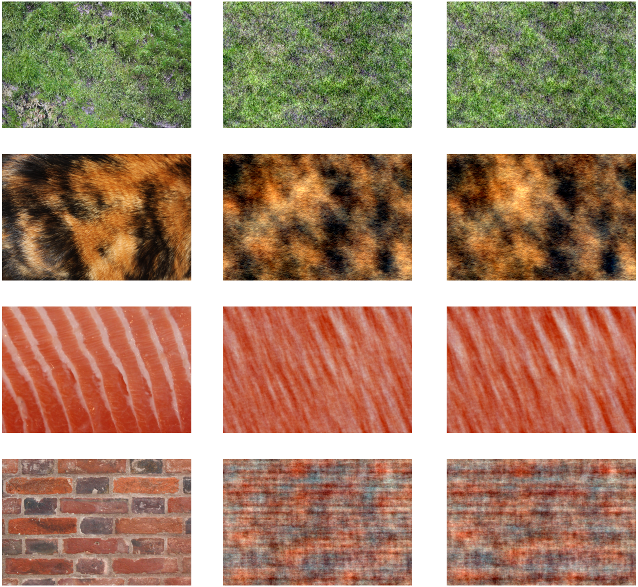
Image h | *ADSN* | *RPN*



- Both algorithms produce visually similar textures.

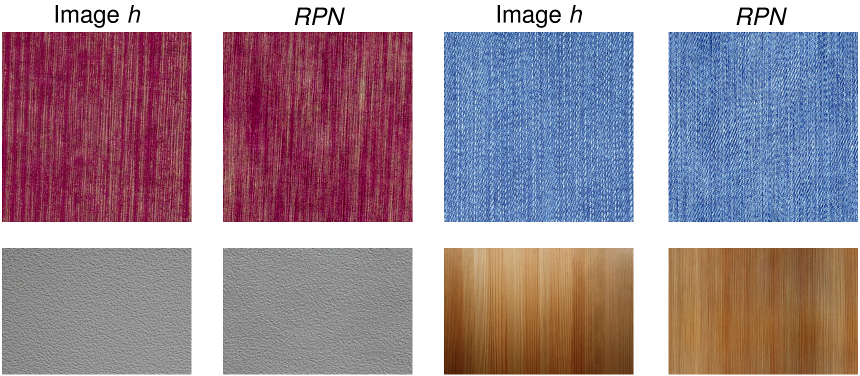
Numerical results: non random phase textures

Image h | *ADSN* | *RPN*



Some other examples of well-reproduced textures...

- We only display the *RPN* result.



- Much more examples of success and failures on the IPOL webpage: `http://www.ipol.im/pub/algo/ggm_random_phase_texture_synthesis/`

Conclusion

Summary:

- *Random phase noise* and *asymptotic discrete spot noise* have been mathematically defined and theoretically compared.
- Both corresponding texture synthesis algorithms are fast, visually stable, and produce visually similar results.
- Both algorithms reproduce relatively well a certain class of textures: the micro-textures.

Limitations:

- The models are limited to a restrictive class of textures.
- The algorithms are not robust to non stationarities, perspective effects, ...
- The method is global: the whole texture image has to be computed (in constrast with noise models from computer graphics).

Outline

Classic texture synthesis algorithms

There is a crucial need for the implementation and study of classic texture synthesis algorithms.

- Easily compare different algorithms.
- Better understand certain parameters of the methods.

Perceptually-based statistical modeling of textures:

- Heeger-Bergen algorithm [Heeger and Bergen, 1995] 823 citations

Non parametric sampling:

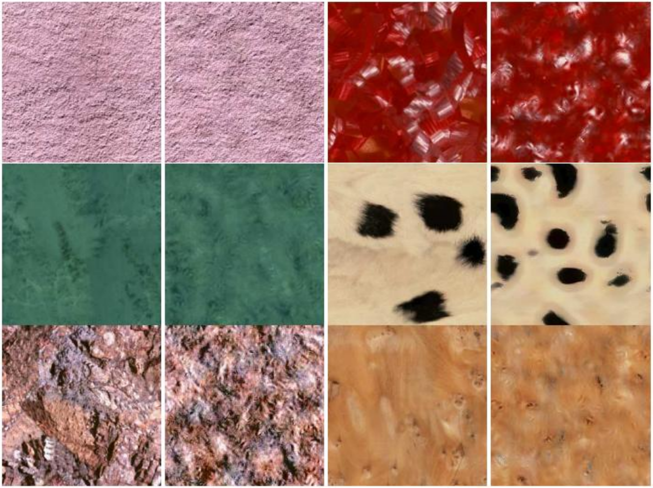
- Efros-Leung algorithm [Efros and Leung, 1999] 1295 citations
- Wei-Levoy algorithm [Wei and Levoy, 2000] 867 citations

From Google Scholar

Heeger-Bergen algorithm [Heeger and Bergen, 1995]

Statistical constraints:

- Histogram of colors.
- Histogram of wavelet coefficients at each scale.



Efros-Leung algorithm [Efros and Leung, 1999]

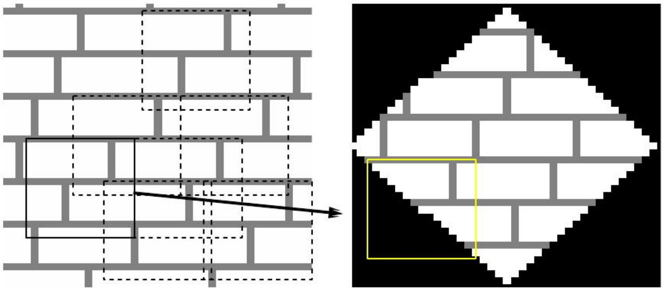
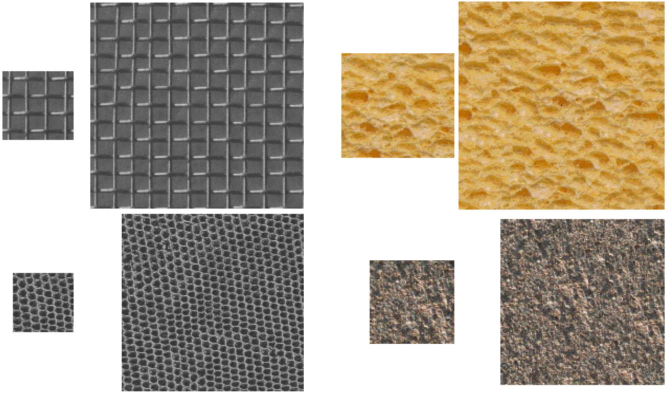


Figure 1. Algorithm Overview. Given a sample texture image (left), a new image is being synthesized one pixel at a time (right). To synthesize a pixel, the algorithm first finds all neighborhoods in the sample image (boxes on the left) that are similar to the pixel's neighborhood (box on the right) and then randomly chooses one neighborhood and takes its center to be the newly synthesized pixel.

Efros-Leung algorithm [Efros and Leung, 1999]

Some successful results:



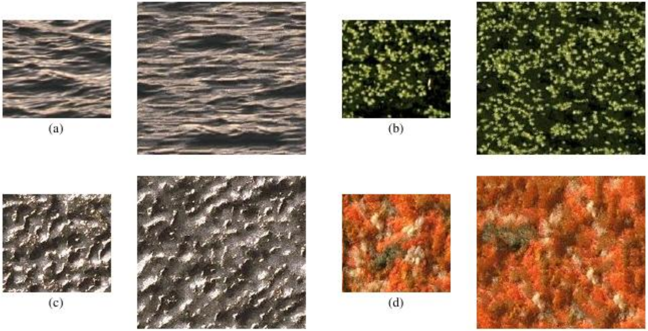
Known problems:

- Can produce “garbage” results.
- Exhaustive searching is really slow.

Wei-Levoy algorithm [Wei and Levoy, 2000]

Similar to Efros-Leung, but:

- Neighborhood always have the same size.
  $\Rightarrow$ Should be easier to accelerate.
- Multi-resolution procedure.
  $\Rightarrow$ More stable than Efros-Leung (?).



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