

- Frame change:  $\hat{\mathbf{X}}_c = \mathbf{R}(\hat{\mathbf{X}} - \mathbf{C})$  with  $\mathbf{X}$  the 3D point in world frame;  $\hat{\mathbf{X}}_c$  the 3D point in camera frame,  $\mathbf{C}$  camera center in world frame,  $\mathbf{R}$  the rotation matrix between camera frame and world frame;  $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3]$  where  $\mathbf{r}_1, \mathbf{r}_2$  and  $\mathbf{r}_3$  the  $X$ -axis,  $Y$ -axis and  $Z$ -axis of world frame represented in camera frame;  $\mathbf{R} = \begin{bmatrix} \mathbf{r}'_1 \\ \mathbf{r}'_2 \\ \mathbf{r}'_3 \end{bmatrix}$  where  $\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{r}'_3$  the  $X$ -axis,  $Y$ -axis and  $Z$ -axis of camera frame represented in world frame.
- Projection from 3D to 2D:

$$\begin{aligned} \mathbf{x}_c &= \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{\mathbf{X}}_c = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} [\mathbf{I} \mid -\mathbf{C}] \begin{pmatrix} \hat{\mathbf{X}} \\ 1 \end{pmatrix} \quad (1) \\ &= \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} [\mathbf{I} \mid -\mathbf{C}] \mathbf{X} \end{aligned}$$

with  $\mathbf{X} = (X, Y, Z, 1)^T$  the homogeneous coordinates of 3D point  $\hat{\mathbf{X}} = (X, Y, Z)^T$ ,  $\mathbf{x}_c$  the homogeneous coordinate of the projected 2D point.

- In pixel coordinate:

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} m_x & 0 & x_0 \\ 0 & m_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\cot\theta & 0 \\ 0 & \frac{1}{\sin\theta} & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_c \\ &= \begin{bmatrix} m_x & 0 & x_0 \\ 0 & m_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\cot\theta & 0 \\ 0 & \frac{1}{\sin\theta} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} [\mathbf{I} \mid -\mathbf{C}] \mathbf{X} \\ &= \begin{bmatrix} m_x f & -m_x f \cot\theta & x_0 \\ 0 & \frac{m_y f}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} [\mathbf{I} \mid -\mathbf{C}] \mathbf{X} \\ &= \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} [\mathbf{I} \mid -\mathbf{C}] \mathbf{X} = \mathbf{K} \mathbf{R} [\mathbf{I} \mid -\mathbf{C}] \mathbf{X} = \mathbf{P} \mathbf{X} \quad (2) \end{aligned}$$

with  $m_x$  and  $m_y$  the number of pixels per unit length in the skewed  $x$ -axis direction and the skewed  $y$ -axis direction in the image plane respectively;  $f$  (in millimeter) the focal length of camera;  $x_0$  and  $y_0$  the coordinates of the principal point, represented in the skewed image frame in pixels;  $s$  the skewness factor which is 0 when the pixel is rectangle;  $\theta$  the skewness angle between two sides of image CCD plane.

- Calibration matrix:

$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} m_x f & -m_x f \cot\theta & x_0 \\ 0 & \frac{m_y f}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

- Projection matrix:  $\mathbf{P} = \mathbf{KR}[\mathbf{I} \mid -\mathbf{C}]$
- Line equation:  $ax+by+c = 0$  can be written as in homogeneous coordinate  $\mathbf{x}^T \mathbf{l} = 0$  with point as  $\mathbf{x} = (x, y, 1)^T$  and line as  $\mathbf{l} = (a, b, c)^T$ .
- The intersection point of two lines  $\mathbf{l} = (a, b, c)^T$  and  $\mathbf{l}' = (a', b', c')^T$  is  $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$  with  $\times$  the cross product.
- The line passing through two points  $\mathbf{x}$  and  $\mathbf{x}'$  has the form  $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$ .
- Homogeneous points  $\mathbf{x} = (x_1, x_2, x_3)^T$  is equivalent to inhomogeneous point  $(x_1/x_3, x_2/x_3)^T$  ( $x_3 \neq 0$ ). Points  $\mathbf{x}$  ( $x_3 = 0$ ) are at infinity. All the points at infinity lie on the line at infinity  $\mathbf{l}_\infty = (0, 0, 1)^T$ .
- 2D projective transformation (homography)  $\mathbf{H}$

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (4)$$

- Epipolar constraint:  $\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0$  with  $\mathbf{F}$  the fundamental matrix
- Given a point  $\mathbf{x}$  in the left image, the corresponding epipolar line in the right image is  $\mathbf{l}' = \mathbf{F} \mathbf{x}$ . Given a point  $\mathbf{x}'$  in the right image, the corresponding epipolar line in the left image is  $\mathbf{l} = \mathbf{F}^T \mathbf{x}'$
- Epipoles:  $\mathbf{F} \mathbf{e} = \mathbf{0}$  and  $\mathbf{F}^T \mathbf{e}' = 0$ .