LSD: a Line Segment Detector

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Outline

- What is LSD, demo
- Overview
- Theoretical background
- Details of the algorithm
- Projects proposed

LSD

- LSD is a Line Segment Detector
- It is based in Burns, Hanson, and Riseman method
- It uses a false detection control based on Desolneux, Moisan, & Morel's theory.
- LSD is fast, produces precise results, and controls false detections.

Resources

Google: lsd + grompone

Google: lsd + morel

www.ipol.im \rightarrow LSD: A LINE SEGMENT DETECTOR









Overview

Gradient and Level-Line Field





image

level-line field

LSD in 4 steps

- 1. Compute the level-line field
- 2. Partition the image into groups of connected pixels that share the same level-line angle up to a certain tolerance
- 3. Find rectangular approximations
- 4. Validation



Line-Support Regions

A group of connected pixels that share the same level-line angle up to a certain tolerance τ .





Rectangular Approximation of Regions



- Pixel's mass is proportional to the gradient modulus
- Region's center of mass rectangle's center
- First inertia axis of the region \longrightarrow rectangle's angle
- Length and width to envelope most of region's mass

Validation



Helmholtz Principle



There is no perception on noise.

A Contrario Detection [Desolneux, Moisan, Morel]

Structure is detected as outliers of a noise model H_0 :



Non-Structured Level-Line Orientations:

- angles are independent random variables
- uniformly distributed in $[0, 2\pi]$

More precisely: an observed geometric structure becomes meaningful when the expectation of its number of occurrences is very small in the non-structured data model.

Aligned Point

A point whose level-line angle is equal to the rectangle angle up to a certain tolerance τ .



k(r, i) is the number of aligned points of rectangle r in image i. n(r) is the total number of pixels in the rectangle r. In the example, k(r, i) = 8 and n(r) = 27.

Meaningful Rectangle

Given a rectangle r with k(r, i) observed aligned points, we define

$$NFA(r, i) = N_{test} \cdot P_{H_0}[k(r, l) \ge k(r, i)]$$

where: *I* is a random image on H_0 , N_{test} is the number of tests.

NFA(r, i) is the expected number of event as good as (r, i) in H_0 . When NFA(r, i) is large: a common event in H_0 and not meaningful. When NFA(r, i) is small: a rare event in H_0 and probably meaningful.

A rectangle with NFA(r, i) $\leq \varepsilon$ is called ε -meaningful rectangle.

Number of Tests



 $N_{test} = N^5$

Probability term

In H_0 , the probability that a pixel is an aligned point is

$$o = \frac{\tau}{\pi}.$$

Because of the independence in H_0 , k(r, I) follows a binomial distribution. Then,

$$P_{H_0}[k(r, l) \ge k(r, i)] = B(n(r), k(r, i), p)$$

where B(n, k, p) is the tail of the binomial distribution:

$$B(n,k,p) = \sum_{j=k}^{n} \binom{n}{j} p^{j} (1-p)^{n-j}$$

NFA

The final expression for the Number of False Alarms for a rectangle is:

$$NFA(r,i) = N^5 \cdot \sum_{j=k(r,i)}^{n(r)} \binom{n(r)}{j} p^j (1-p)^{n(r)-j}$$

Theorem

$$E_{H_0}\left[\sum_{r\in\mathcal{R}}\mathbb{1}_{\mathrm{NFA}(r,l)\leq\varepsilon}\right]\leq\varepsilon$$

where *E* is the expectation operator, $\mathbb{1}$ is the indicator function, \mathcal{R} is the set of rectangles considered, and *I* is a random image in H_0 .

The theorem states that the average number of ε -meaningful rectangles on the a contrario model H_0 images is less than ε .

In other words, it shows that LSD satisfies the Helmholtz principle.

Proof

We define $\hat{k}(r)$ as

$$\hat{k}(r) = \min\left\{n \in \mathbb{N}, \ P_{H_0}[k(r,l) \ge n] \le \frac{\varepsilon}{N^5}\right\}.$$

Then, NFA $(r, i) \le \varepsilon$ is equivalent to $k(r, i) \ge \hat{k}(r)$. Now,

$$E_{H_0}\left[\sum_{r\in\mathcal{R}}\mathbbm{1}_{\mathrm{NFA}(r,l)\leq\varepsilon}\right]=\sum_{r\in\mathcal{R}}P_{H_0}\left[\mathrm{NFA}(r,l)\leq\varepsilon\right]=\sum_{r\in\mathcal{R}}P_{H_0}\left[k(r,l)\geq\hat{k}(r)\right].$$

But, by definition of $\hat{k}(r)$ we know that

$$\mathsf{P}_{\mathsf{H}_0}\left[k(r,l)\geq \hat{k}(r)
ight]\leq rac{arepsilon}{\mathsf{N}^5}$$

and using that $\#\mathcal{R} = N^5$ we get

$$E_{H_0}\left[\sum_{r\in\mathcal{R}}\mathbbm{1}_{\mathrm{NFA}(r,l)\leq\varepsilon}\right]\leq\sum_{r\in\mathcal{R}}\frac{\varepsilon}{N^5}=\varepsilon.$$

$\varepsilon = 1$

The result is not very sensitive to the value of ε .



image

 $\varepsilon = 1$

 $\varepsilon = 0.1$

 $\varepsilon = 0.01$

 $\varepsilon = 1$ means, on average, one false detection per image.

Algorithm Summary

- 1. Partition the image into Line-Support Regions
- 2. For each Line-Support Region:
- 3. Find the Rectangular Approximation
- 4. Compute NFA value
- 5. Rectangles with $NFA \leq 1$ are added to the output.

more examples

















Details of the algorithm

LSD

- 1. Scale the input image to scale S ($\sigma = \Sigma/S$).
- 2. Compute level-lines field.
- 3. List pixels by decreasing gradient magnitude.
- 4. Set STATUS(every pixel) to NOT USED.
- 5. Remove pixels where gradient magnitude $\leq \rho$.
- 6. From the next pixel P in the list with STATUS(P)=NOT USED:
 - 7. Grow region from P of NOT USED connected pixels that share level-line angle, tolerance τ . Mark pixels in the region as USED.
 - 8. Compute the rectangular approximation.
 - 9. Cut region until aligned point density > *D*.
 - 10. Compute NFA value.
 - 11. Try to improve rectangle.
 - 12. If NFA $\leq \varepsilon$, detection!

Parameters S, Σ , ρ , τ , D, and ε .

Input image scaling

Staircase problem:



input image

80% scaling

80% scale, then S = 0.8. Gaussian sub-sampling with $\sigma = \Sigma/S$ Good balance between blur and aliasing: $\Sigma = 0.6$

Compute level-line field

The gradient is computed at (x, y) + (1/2, 1/2) by

$$\begin{array}{c} \begin{array}{c} \begin{array}{c} + \\ - \\ + \end{array} \\ g_x(x,y) = \frac{i(x+1,y)+i(x+1,y+1)-i(x,y)-i(x,y+1)}{2}, \\ \end{array} \\ \\ \begin{array}{c} + \\ + \\ - \\ - \end{array} \\ g_y(x,y) = \frac{i(x,y+1)+i(x+1,y+1)-i(x,y)-i(x+1,y)}{2}. \end{array}$$

The level-line angle is computed as

$$\arctan\left(\frac{g_x(x,y)}{-g_y(x,y)}\right)$$

and the gradient norm as

$$G(x,y)=\sqrt{g_x^2(x,y)+g_y^2(x,y)}.$$

Where i(x, y) is the image value at coordinates (x, y).

Pseudo-ordering of pixels



image

gradient magnitude

Starting from pixels of high gradient magnitude, seed points are near the center of edges.

Sorting cannot be done in linear time. Instead, a pseudo-ordering is performed by classifying pixels in 1024 bin of gradient values.

Gradient threshold



 $\tilde{i} = i + n$ $\nabla \tilde{i} = \nabla i + \nabla n$,

where *n* is the quantization noise.

$$|\text{angle error}| \leq \arcsin\left(\frac{q}{|
abla i|}
ight),$$

where *q* is a bound to $|\nabla n|$. Imposing |angle error| $\leq \tau$ we get

$$\rho = \frac{q}{\sin \tau}$$

q = 2, maximum gradient quantization error in [0, 255] images.

Region Growing

Region's angle:



Recursively, the unused neighbors Q are added if

level-line-angle(
$$Q$$
) – $\theta_{region}\Big|_{\text{mod}2\pi} < \tau$.

 τ = 22.5 degree.

Angle problem

If two line segments for an angle of 180 $-\tau$ we get:

We define the density of aligned points as

$$d = \frac{k}{\operatorname{length}(r) \cdot \operatorname{width}(r)}$$

where k is the number of aligned points in the region.

The region is repeatedly cut until d > D or there are no points left.

D = 0.7 is an empirical value.

Improve rectangle

Before rejecting a region as not meaningful, some variations to the rectangle are tried:

- 1. try finer precisions p
- 2. try to reduce width
- 3. try to reduce one side of the rectangle
- 4. try to reduce the other side of the rectangle
- 5. try even finer precisions

Parameters

- S = 0.8, staircase effect
- $\Sigma = 0.6$, blur/aliasing balance
- $ho = q/\sin au$, q = 2, quantization noise
- au= 22.5 degree, empirical but near optimum
- D = 0.7, empirical
- $\varepsilon = 1$, a contrario framework

Only *D* has an arbitrary value and determines how curves are approximated.

More details: www.ipol.im

Projects

















- 1. "Local Scale Control for Edge Detection and Blur Estimation" by James H. Elder & Steven W. Zucker
- "Multiscale Edge Detection and Fiber Enhancement Using Differences of Oriented Means" by Meirav Galun, Ronen Basri & Achi Brandt

Project 4: Devernay's sub-pixel edge detector



"A Non-Maxima Suppression Method for Edge Detection with Sub-Pixel Accuracy" by Frédéric Devernay

IPOL Publication

- Detailed description of the algorithm
- Good quality code: standard and well commented
- A running demonstration

video

merci