

The background of the slide is a grayscale image of zebrafish embryos, showing various stages of development. The embryos are scattered across the frame, with some appearing as small, dark spots and others as larger, more complex structures. The overall appearance is that of a microscopic view of a zebrafish embryo culture.

# 3D IMAGES of ZEBRAFISH EMBRYO: PDEs based FILTERING and SEGMENTATION

Barbara Rizzi  
DEIS  
University of Bologna  
*Italy*

Supervisor:

Prof. Alessandro Sarti  
DEIS  
University of Bologna  
*Italy*

Joint work with:

Cecilia Zanella  
II Faculty of Engineering  
University of Bologna  
*Italy*

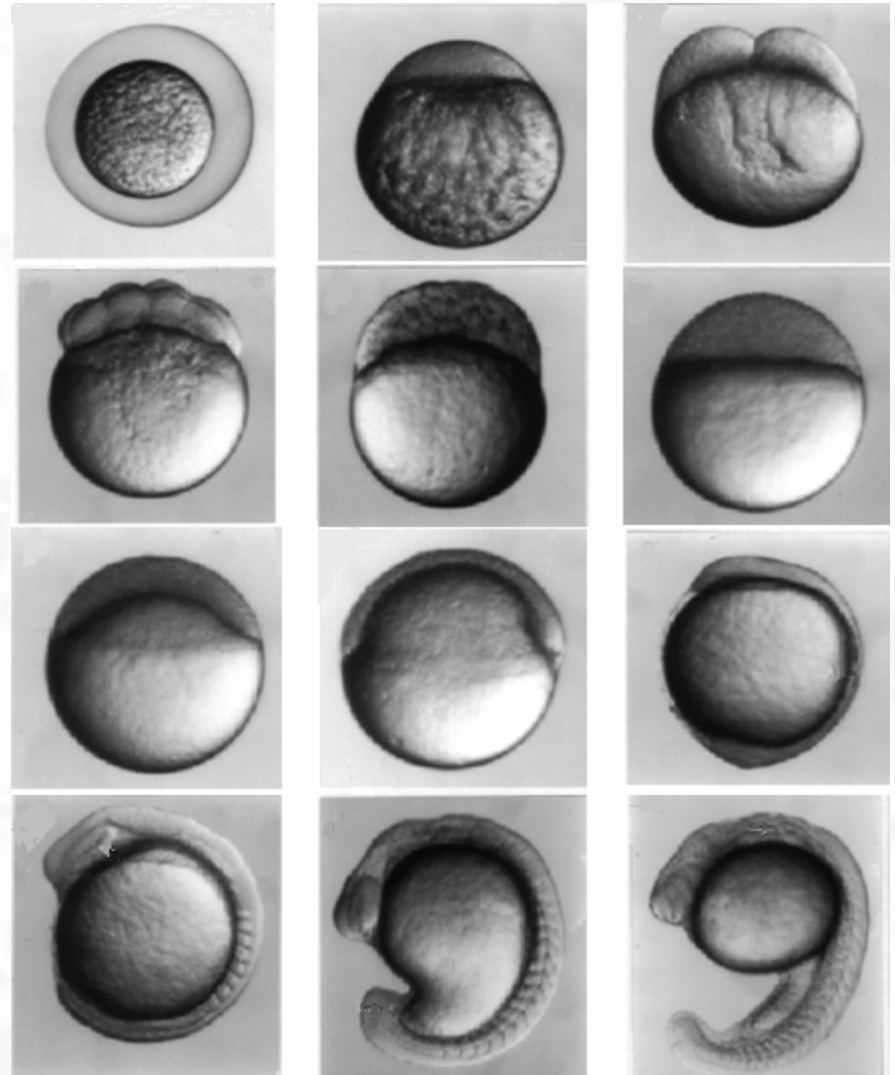
# DATA

- Project EMBRYOMICS FP6-2003-NEST-B-1
- confocal microscopy
- Gif sur Yvette
- nuclei and membranes of zebrafish embryo

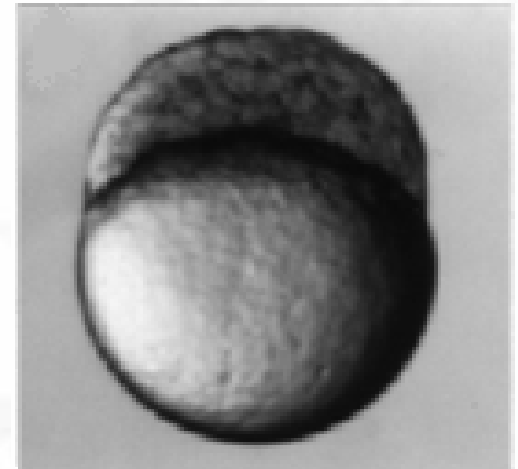
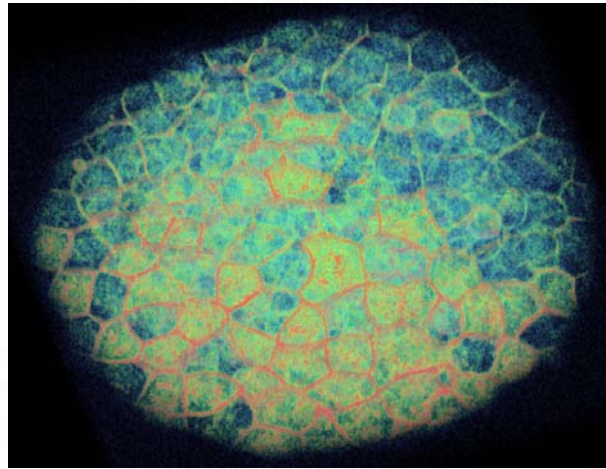
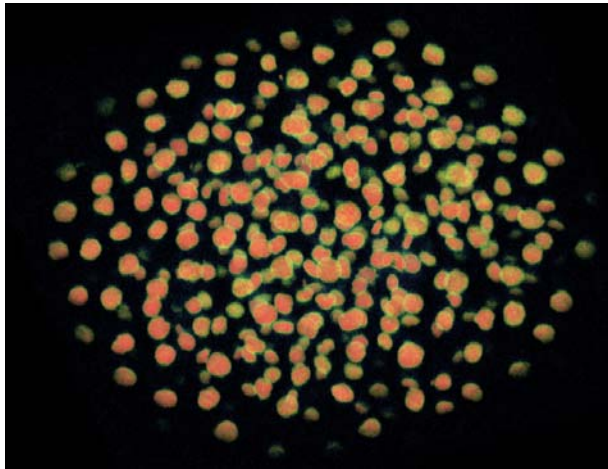


# DATA

- Project EMBRYOMICS FP6-2003-NEST-B-1
- confocal microscopy
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# DATA



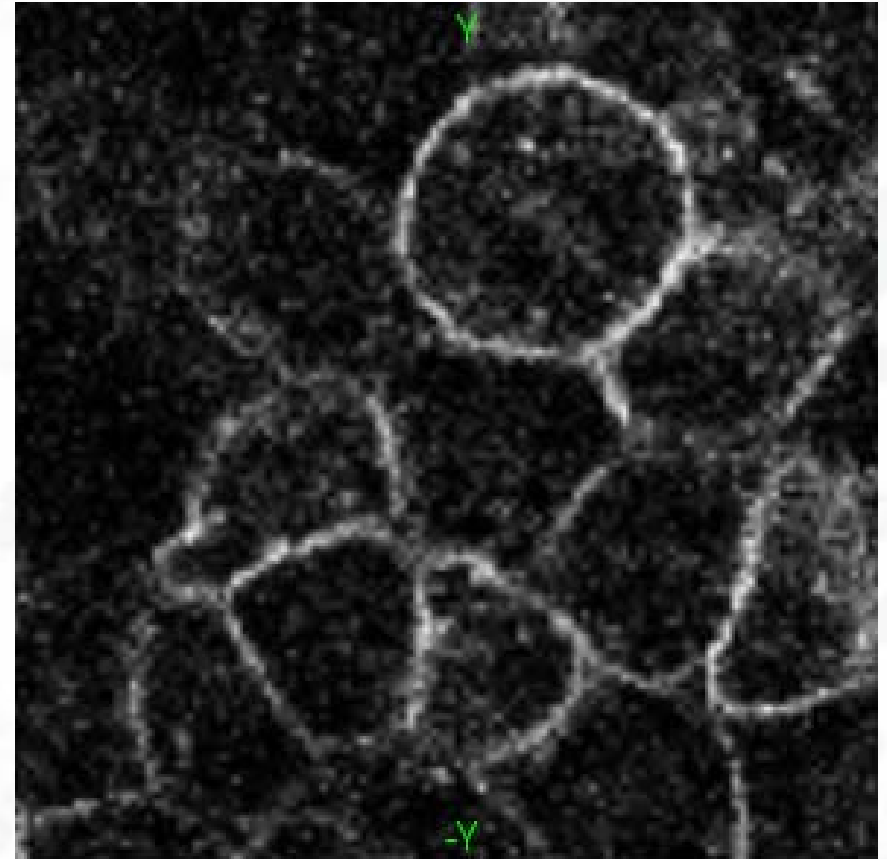
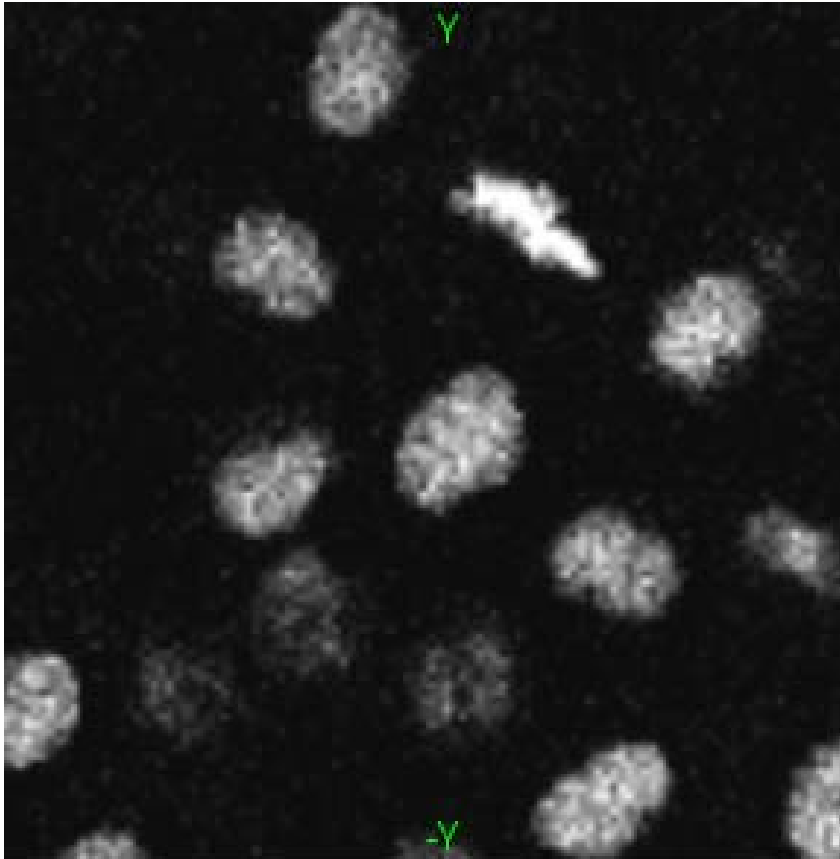
Leica confocal SP2 AOBS

blastula period

5 hours  
development

Size in voxels:	512 x 512 x 30;
Sample spacing:	0.584793 x 0.584793 x 1.048915 $\mu\text{m}^3$ /voxel;
Physical size:	299.41 x 299.41 x 31.47 $\mu\text{m}^3$ ;

# PURPOSE of the work

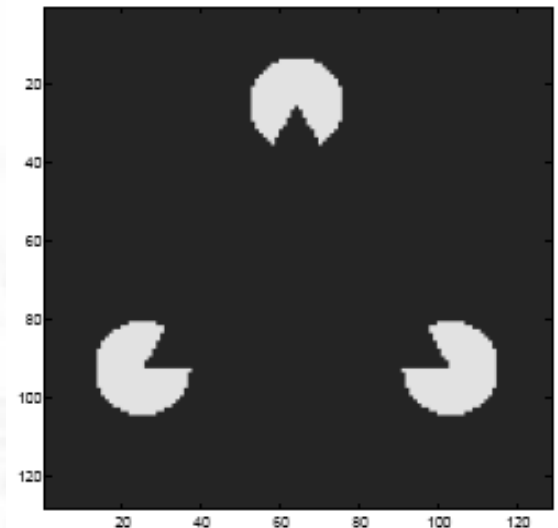
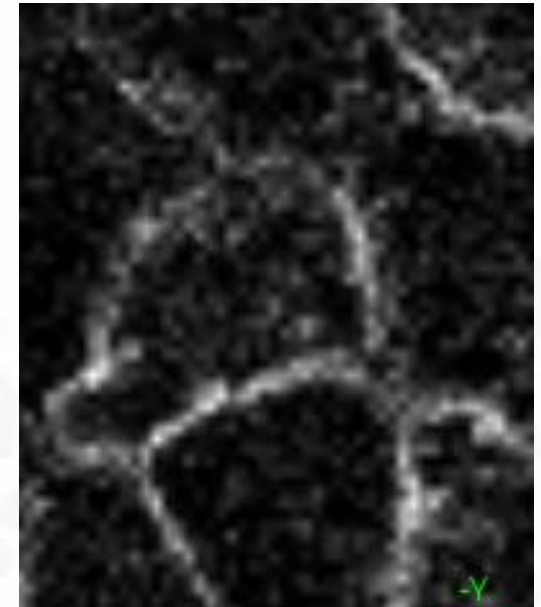


- Filtering: nuclei and membranes
- Segmentation: membranes

- PDEs based methods
- Subjective Surfaces

# SUBJECTIVE SURFACES

- A.Sarti, R.Malladi, J.Sethian (2000)
- A.Sarti, G.Citti (2000)
- Segmentation of objects with missing boundaries (membranes)
- Segmentation of apparent or subjective contours
- Kanisza Triangle
- Point of view of the observer
- Reference surface



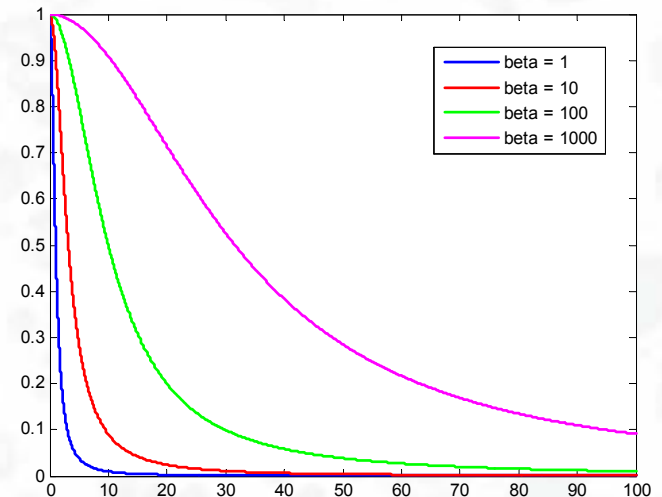
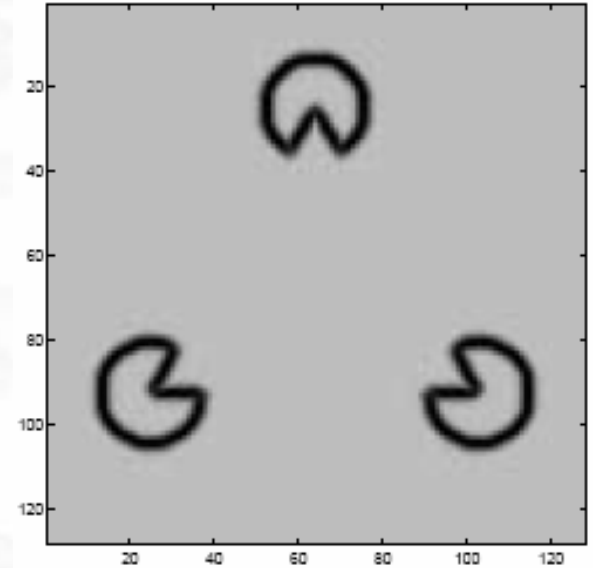
# LOW LEVEL FEATURES EXTRACTION

- Edge detector  $g$ :

$$g(x, y) = \frac{1}{1 + |G_\sigma(x, y) * \nabla I(x, y, 0)|^2 / \beta}$$

$$G_\sigma(\xi) = \frac{\exp(-(\xi/\sigma)^2)}{\sigma\sqrt{\pi}}$$

Gaussian Kernel:  
minimal size of details



# SUBJECTIVE SURFACES

- Selection of the point of view
- Construction of a reference surface

$l : (x, y) \rightarrow l(x, y)$  real positive function in  $\Omega \subset \mathbb{R}^2$

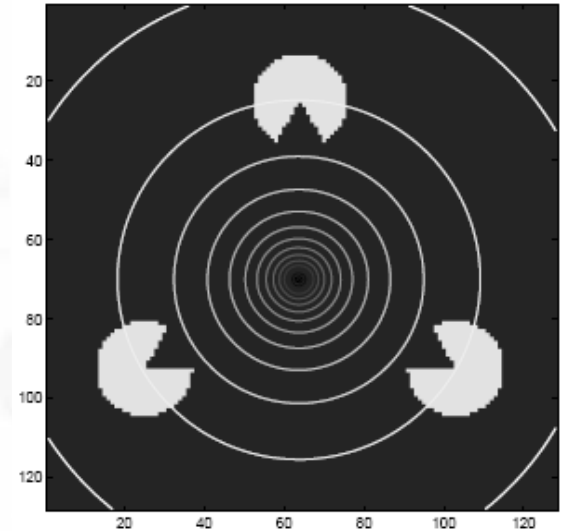
$\Phi$ : distance or peak function in  $\Omega$

$S := \text{graph} \{ \Phi \} := \{ (x, y, \Phi(x, y)) : (x, y) \in \Omega \}$

$S := 2D$  manifold embedded in  $(\mathbb{R}^3, h)$

$$h = \begin{pmatrix} g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & \frac{g}{\varepsilon} \end{pmatrix}$$

Riemannian metric  
in  $(\mathbb{R}^3, h)$





# SUBJECTIVE SURFACES

- Selection of the point of view
- Construction of a reference surface

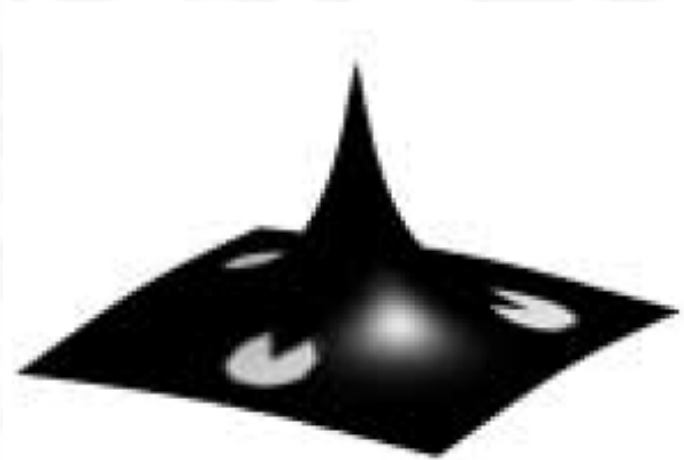
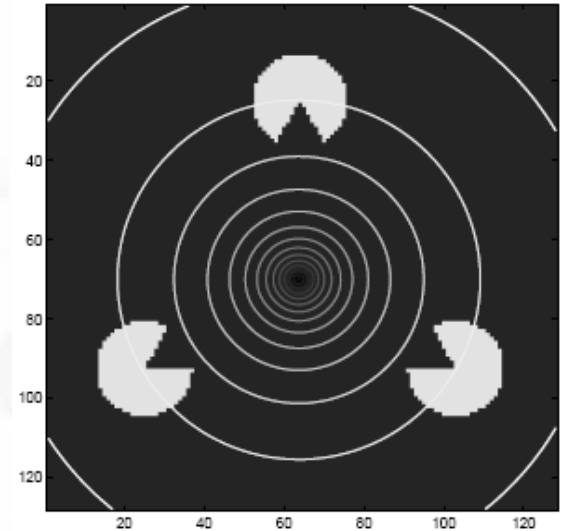
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$S := 2D$  manifold embedded in  $(\mathbb{R}^3, h)$

$$A_g = \int_S \frac{g(x, y)}{\sqrt{\varepsilon}} \sqrt{\varepsilon + \Phi_x^2 + \Phi_y^2} dx dy$$



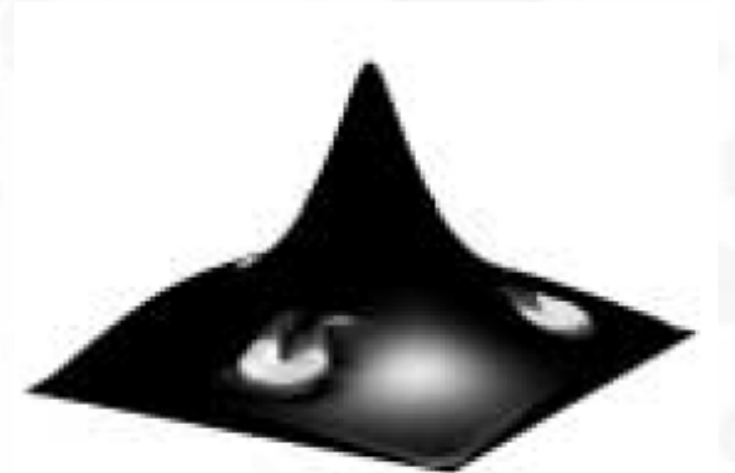
# SUBJECTIVE SURFACES

- Selection of the point of view
- Construction of a reference surface
- Evolution of the surface with mean curvature flow

$$\frac{\partial S}{\partial t} = H \cdot \nu$$

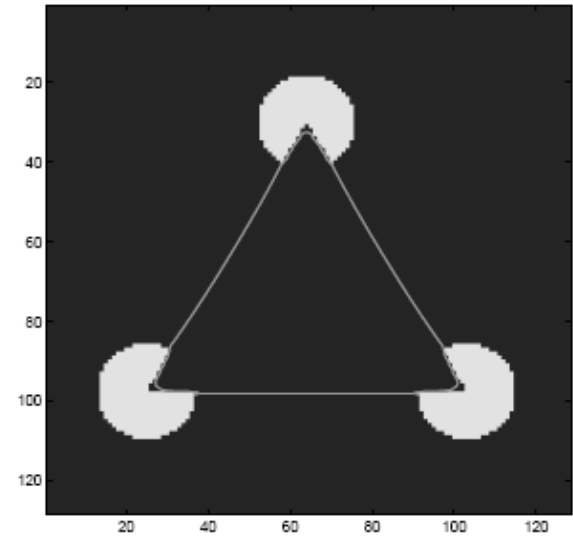
-H: mean curvature of  
S in  $(\mathbb{R}^3, h)$

- $\nu$ : inner normal of S in  
 $(\mathbb{R}^3, h)$



# SUBJECTIVE SURFACES

- Selection of the point of view
- Construction of a reference surface
- Evolution of the surface with mean curvature flow
- Surface attracted by the existing boundaries
- Piecewise constant solution: minimum of  $A_g$
- Segmentation



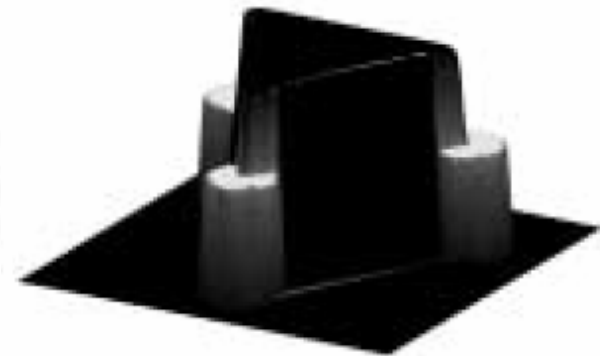
# MODEL EQUATION

$$H = g \frac{(\varepsilon + \Phi_x^2)\Phi_{yy} + (\varepsilon + \Phi_y^2)\Phi_{xx} - 2\Phi_x\Phi_y\Phi_{xy}}{(\varepsilon + \Phi_x^2 + \Phi_y^2)^{3/2}} + \frac{g_x\Phi_x + g_y\Phi_y}{(\varepsilon + \Phi_x^2 + \Phi_y^2)^{1/2}}$$

$$\frac{\partial S}{\partial t} = H \cdot \nu \longrightarrow \frac{\partial \Phi}{\partial t} = H \cdot |\nabla \Phi|$$

-H: mean curvature of  
S in  $(\mathbb{R}^3, h)$

- $\nu$ : inner normal of S in  
 $(\mathbb{R}^3, h)$



MINIMUM of  $A_g$

# MODEL EQUATION

$$\frac{\partial \Phi}{\partial t} = g \cdot \frac{(\varepsilon + \Phi_x^2)\Phi_{yy} + (\varepsilon + \Phi_y^2)\Phi_{xx} - 2\Phi_x\Phi_y\Phi_{xy}}{(\varepsilon + \Phi_x^2 + \Phi_y^2)^{3/2}} \cdot |\nabla \Phi| + g_x\Phi_x + g_y\Phi_y$$

K

$$\frac{\partial \Phi}{\partial t} = g \cdot K \cdot |\nabla \Phi| + \nabla g \cdot \nabla \Phi \quad \text{SEGMENTATION}$$

$$\frac{\partial I}{\partial t} = g \cdot K \cdot |\nabla I| + \nabla g \cdot \nabla I \quad \text{FILTERING}$$

- R.Malladi, J. Sethian, B. Vemuri (1995);
- V.Caselles, R. Kimmel, G.Sapiro (1997);
- A.Sarti, C.Solòrzano, S.Lockett, R.Malladi (2000);

# FILTERING

- Mean curvature

$$\begin{cases} I_t = K \cdot |\nabla I| \\ I(x, y, z, t) = \bar{I} \\ I(x, y, z, 0) = I_0(x, y, z) \end{cases}$$

- 3D + time space

$$\begin{cases} I_t = clt \cdot (g \cdot K \cdot |\nabla I| + \nabla g \cdot \nabla I) \\ I(x, y, z, t) = \bar{I} \\ I(x, y, z, 0) = I_0(x, y, z) \end{cases}$$

- Geodesic curvature

$$\begin{cases} I_t = g \cdot K \cdot |\nabla I| + \nabla g \cdot \nabla I \\ I(x, y, z, t) = \bar{I} \\ I(x, y, z, 0) = I_0(x, y, z) \end{cases}$$

clt : = Curvature Lambertian  
Trajectory

# 3D + TIME

$$\begin{cases} I_t = clt \cdot (g \cdot K \cdot |\nabla I| + \nabla g \cdot \nabla I) \\ I(x, y, z, t) = \bar{I} \\ I(x, y, z, 0) = I_0(x, y, z) \end{cases}$$

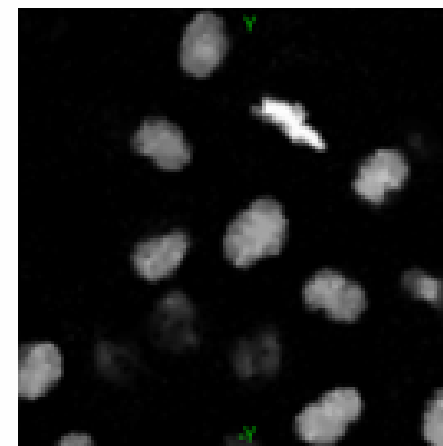
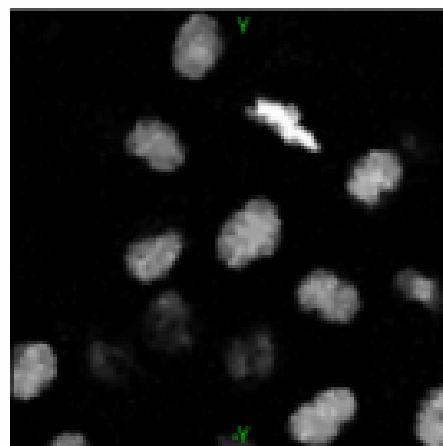
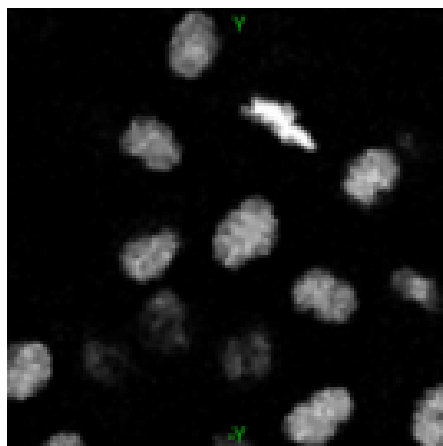
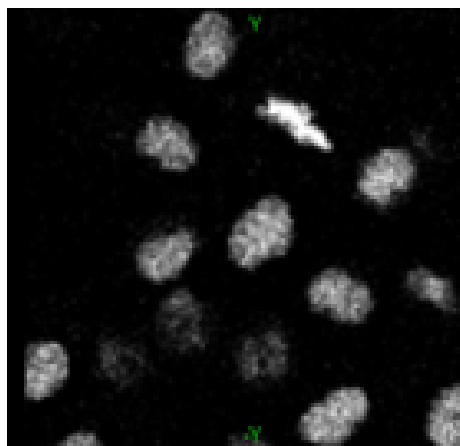
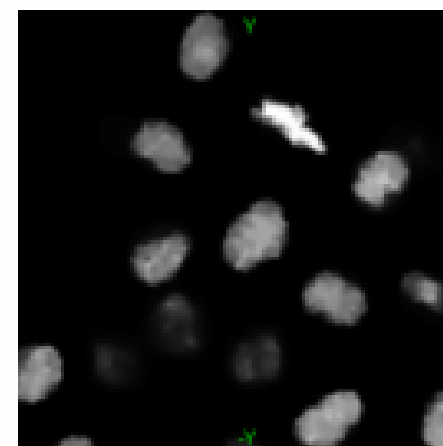
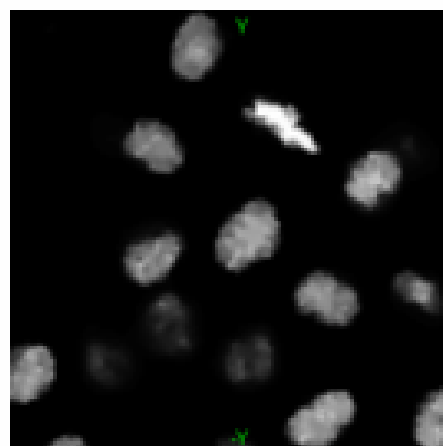
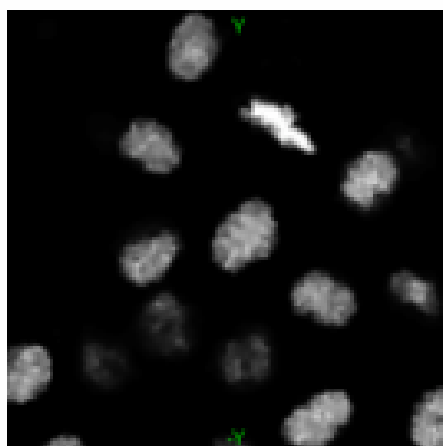
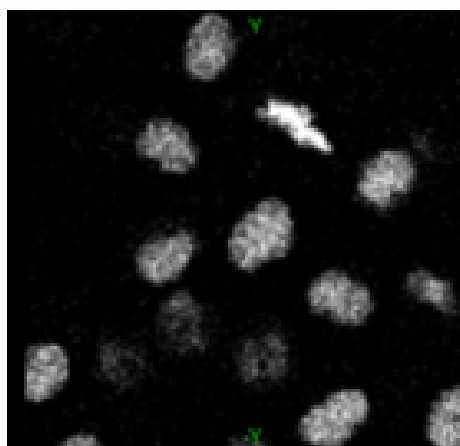
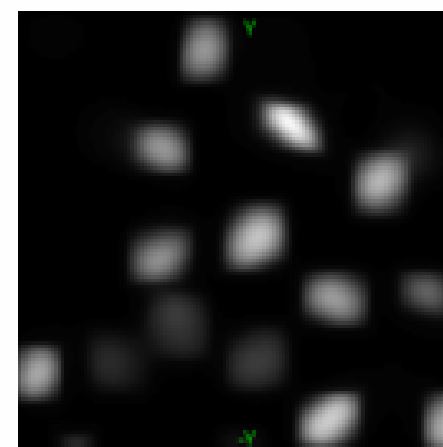
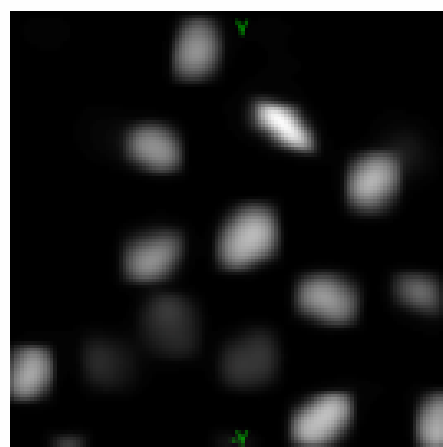
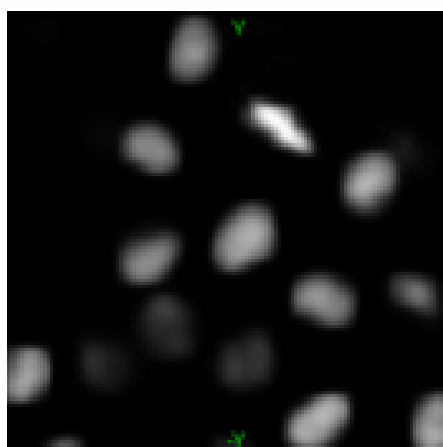
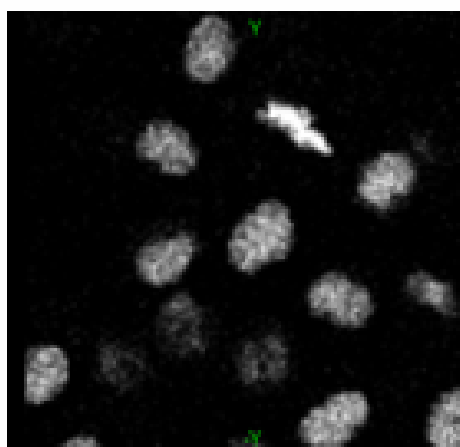
- F.Guichard (1994)
  - Lambertian structures
  - motion locally Galilean in time
  - measure of coherence in time
- for moving structures

$$clt(I) = \min_{w_1, w_2} \frac{1}{(\Delta\tau)^2} \left( \left| \langle \nabla I, w_1 - w_2 \rangle \right| + \left| I(x - w_1, \tau - \nabla\tau) - I(x, \tau) \right| + \left| I(x + w_2, \tau + \nabla\tau) - I(x, \tau) \right| \right)$$

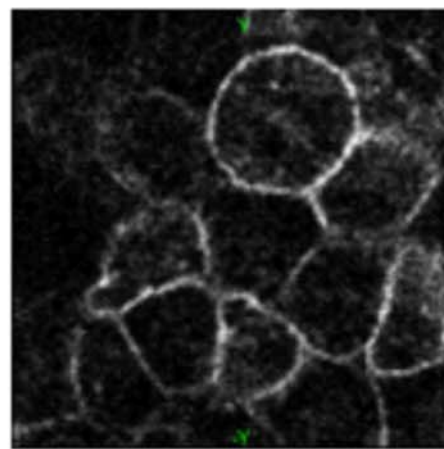
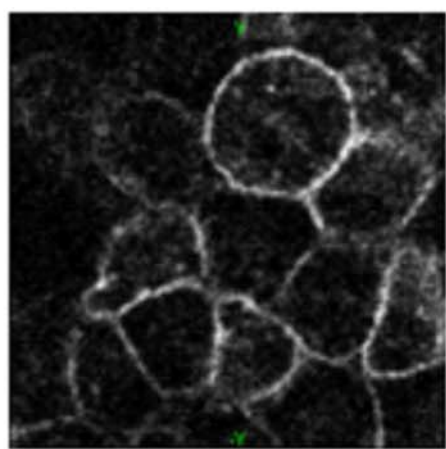
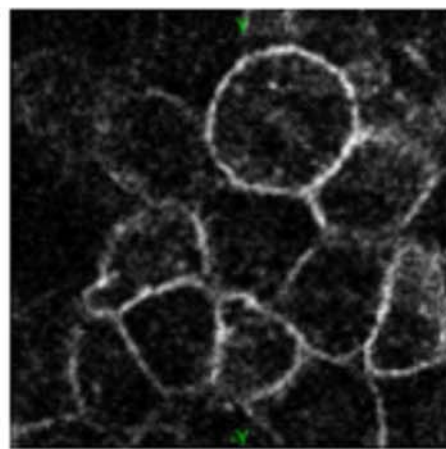
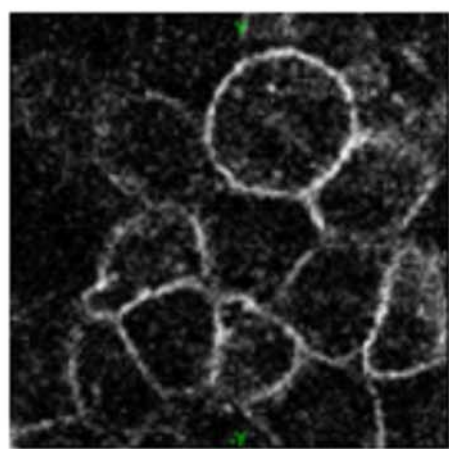
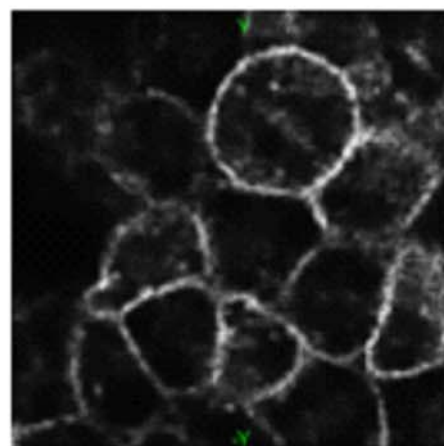
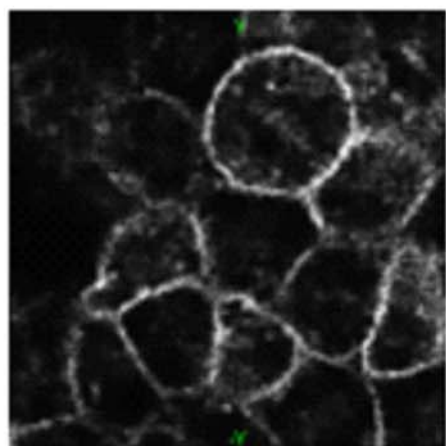
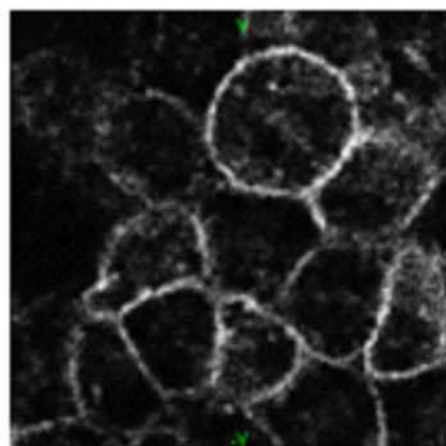
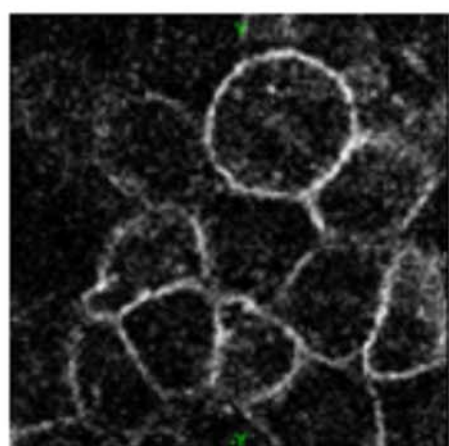
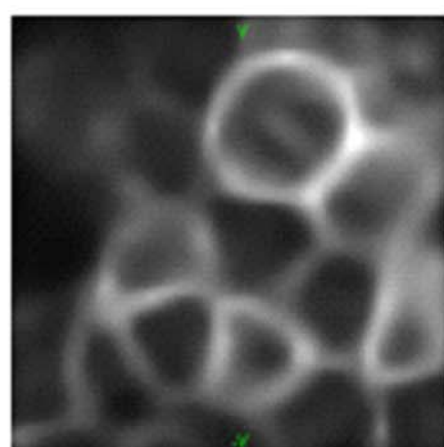
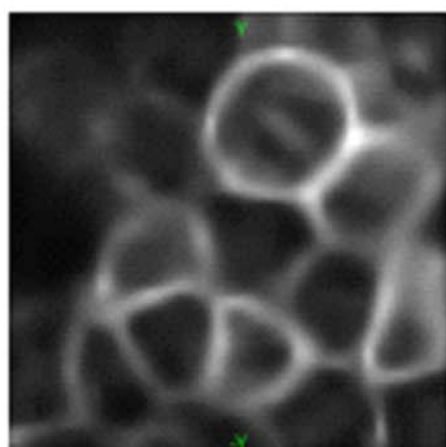
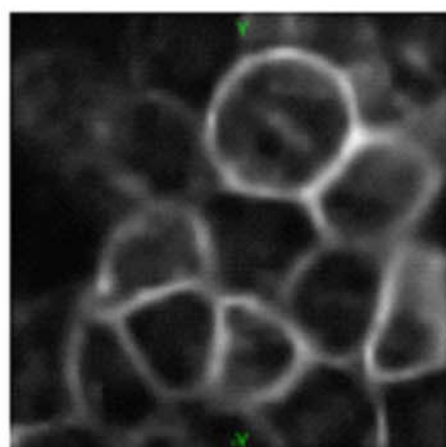
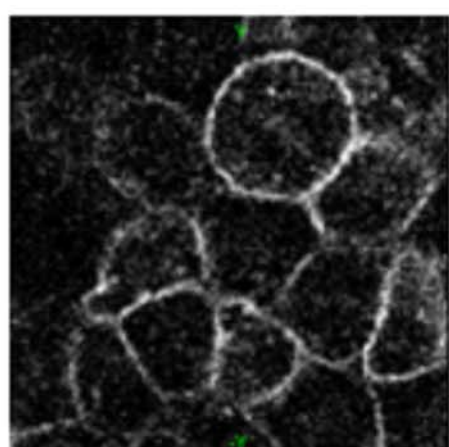
- $w_1, w_2$  arbitrary vectors in  $\mathbb{R}^3$
- $\Delta\tau$  time increment

$\Delta\tau + \tau$  : next frame

$\Delta\tau - \tau$  : previous frame







# GEODESIC CURVATURE FILTERING:

$$\begin{cases} I_t = g \cdot K \cdot |\nabla I| + \nabla g \cdot \nabla I \\ I(x, y, z, t) = \bar{I} \\ I(x, y, z, 0) = I_0(x, y, z) \end{cases}$$

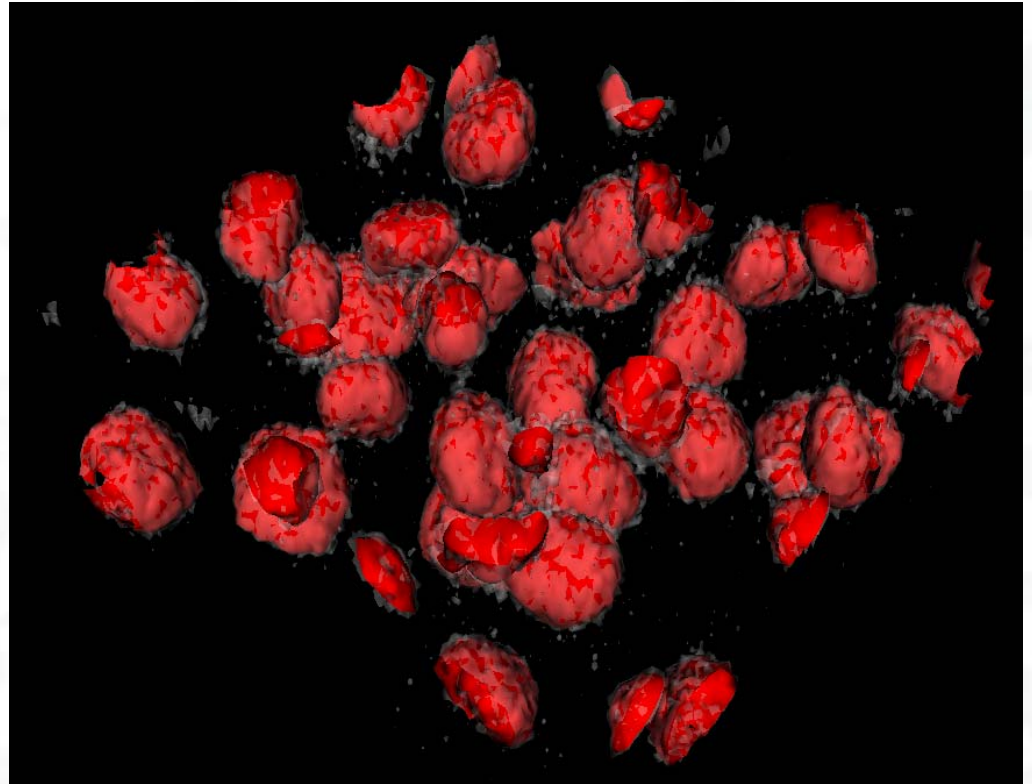
$$\varepsilon = 0.1;$$

$$\sigma = 0.05;$$

$$\beta = 100;$$

$$dt = 0.01;$$

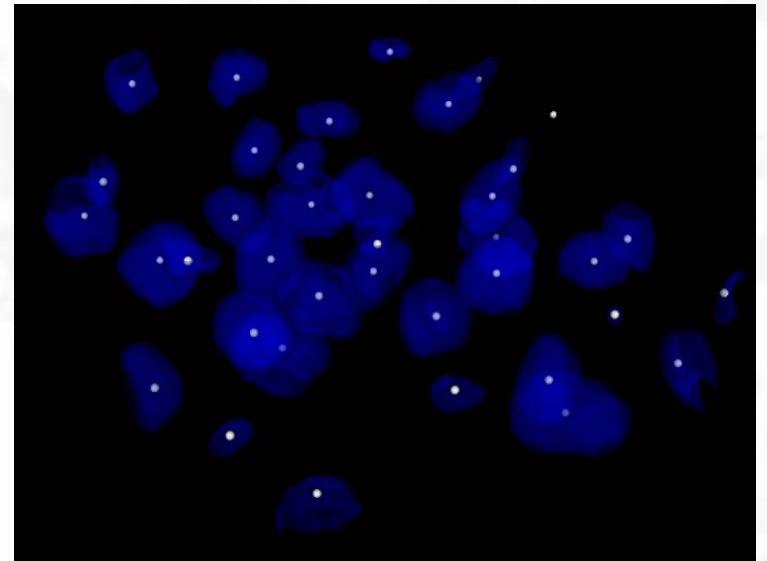
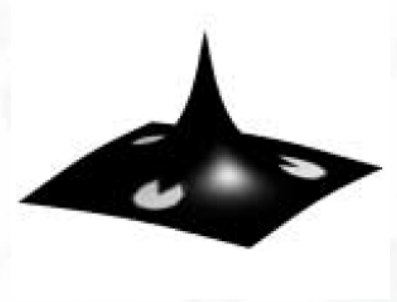
500 scale steps;



# SEGMENTATION

## SELECTION OF THE POINT OF VIEW

- a point in the center of every membranes
- nuclei inside membranes
- centers of nuclei
- filtered data: isosurface centers



# SEGMENTATION

$$\frac{\partial \Phi}{\partial t} = \mu \cdot g \cdot K \cdot |\nabla \Phi| + \nu \cdot \nabla g \cdot \nabla \Phi$$

$$\Phi_0 = -\alpha/D, \quad \alpha = 255;$$

50000 scale steps with  $dt = 0.01$ ;

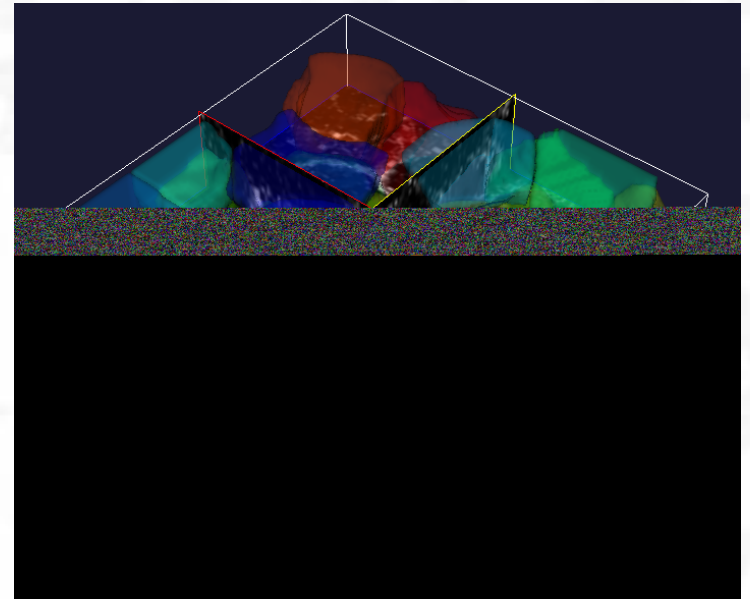
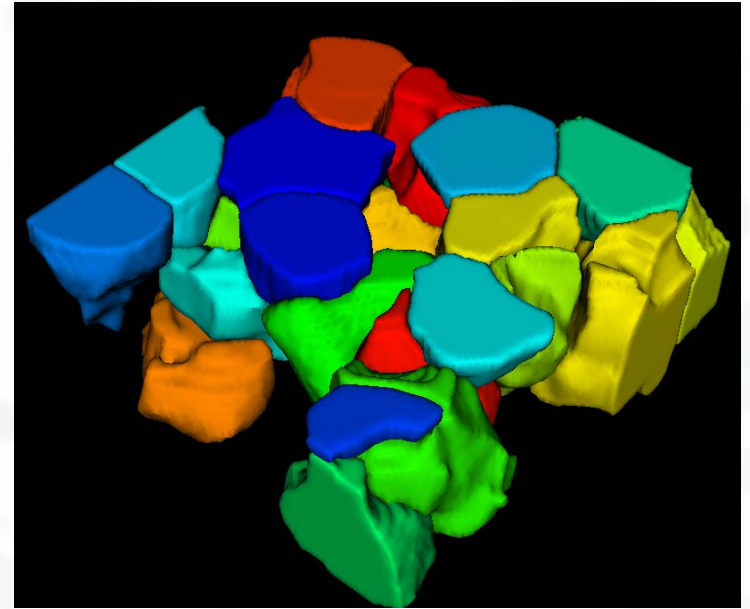
$\varepsilon = 1.0$  first 10000 scale steps;

$\varepsilon = 1.0e^{-6}$  next 40000 scale steps;

$$\sigma = 1;$$

$$\mu = 0.1;$$

$$\nu = 10;$$



# WORK IN PROGRESS....

## FINITE VOLUME METHOD

- speed improvement
- semi-implicit scheme

## VALIDATION

- Manual segmentation
- Gold standard
- Distance between the gold and the original
- Distance between the gold and the filtered



THANK YOU

Barbara Rizzi  
Department of Electronic, Computer  
Science and Systems  
University of Bologna  
*Italy*