Seuils de perception

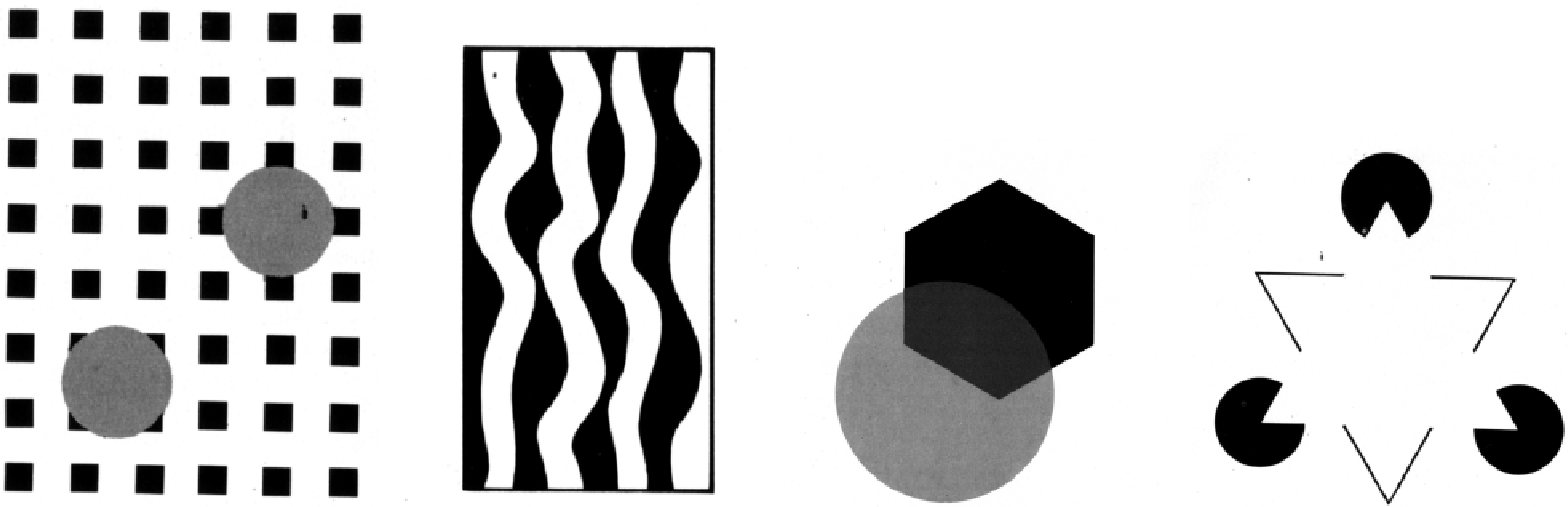
Rafael Grompone
José Lezama

From Gestalt Theory to Image Analysis

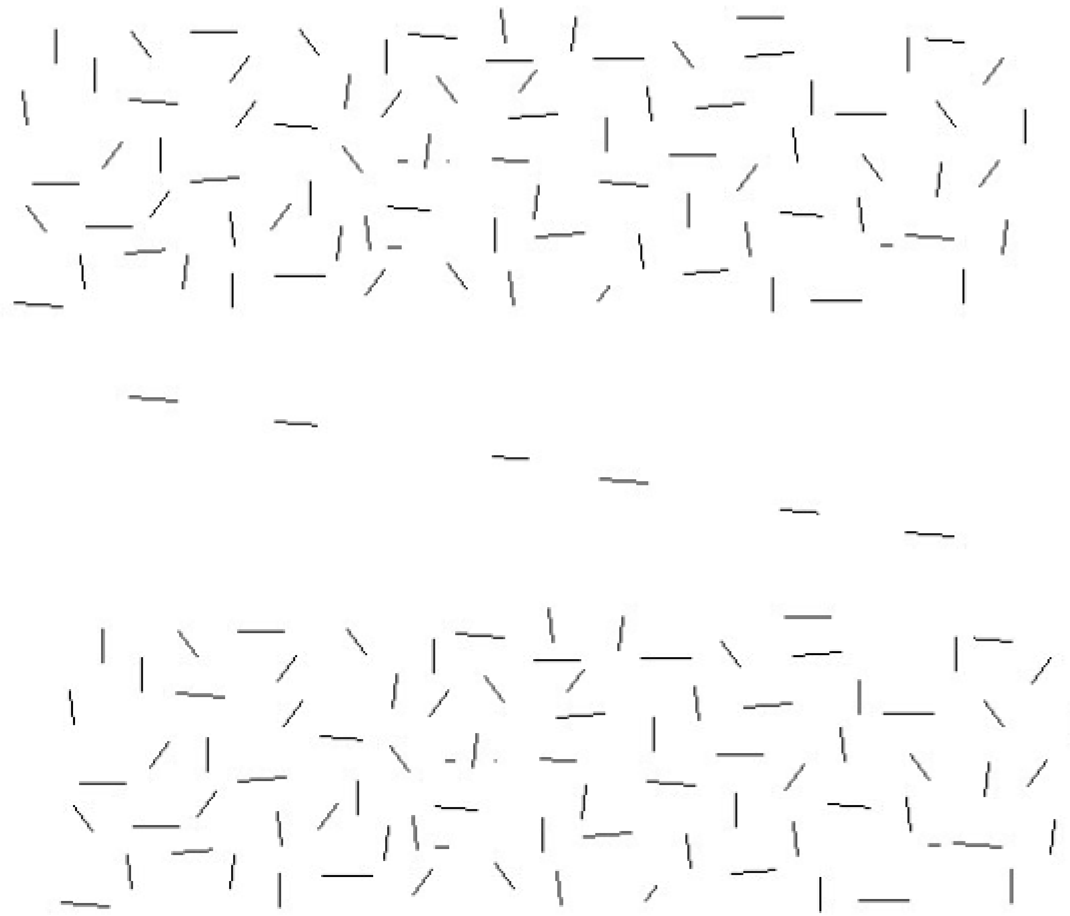
A Probabilistic Approach

Agnès Desolneux
Lionel Moisan
Jean-Michel Morel

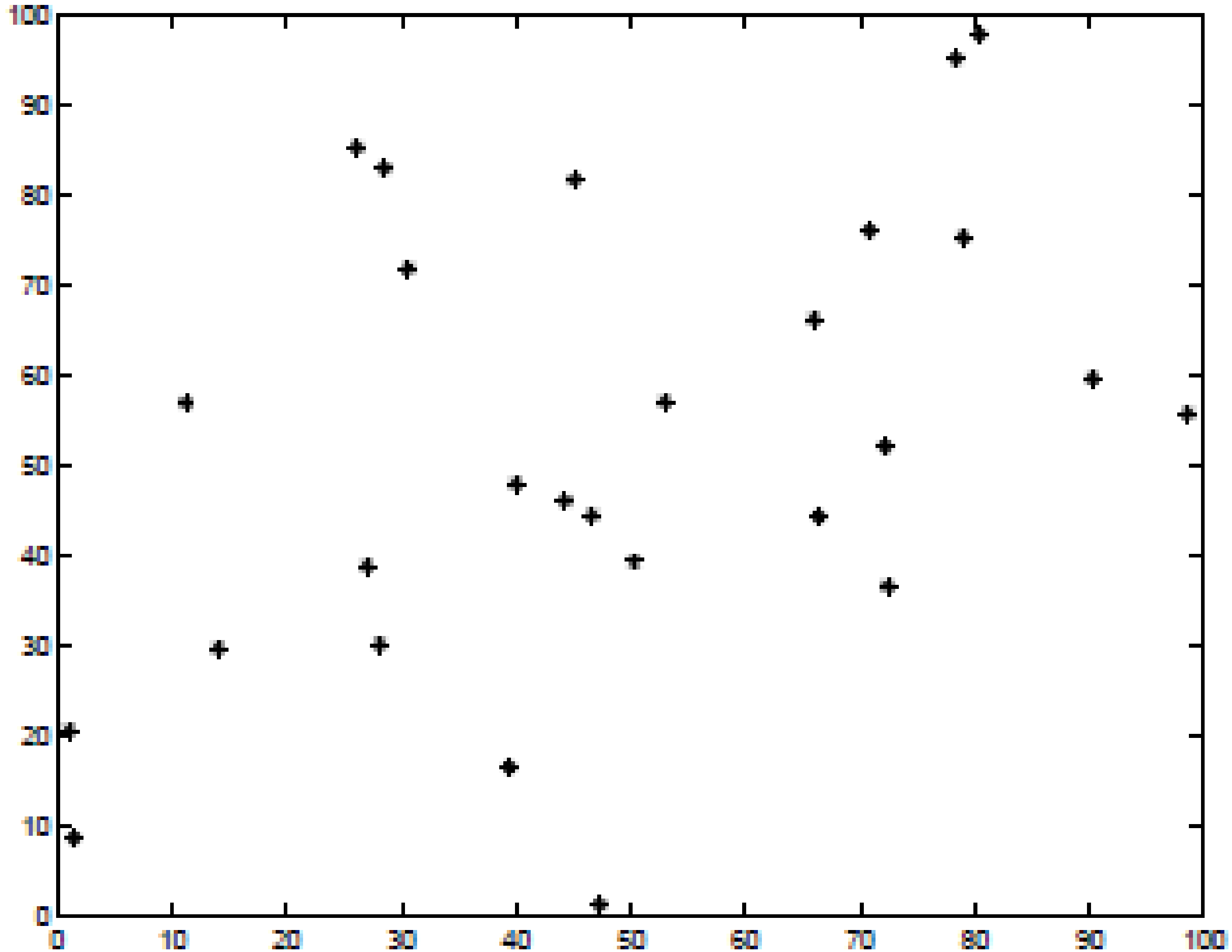
Gestalt School



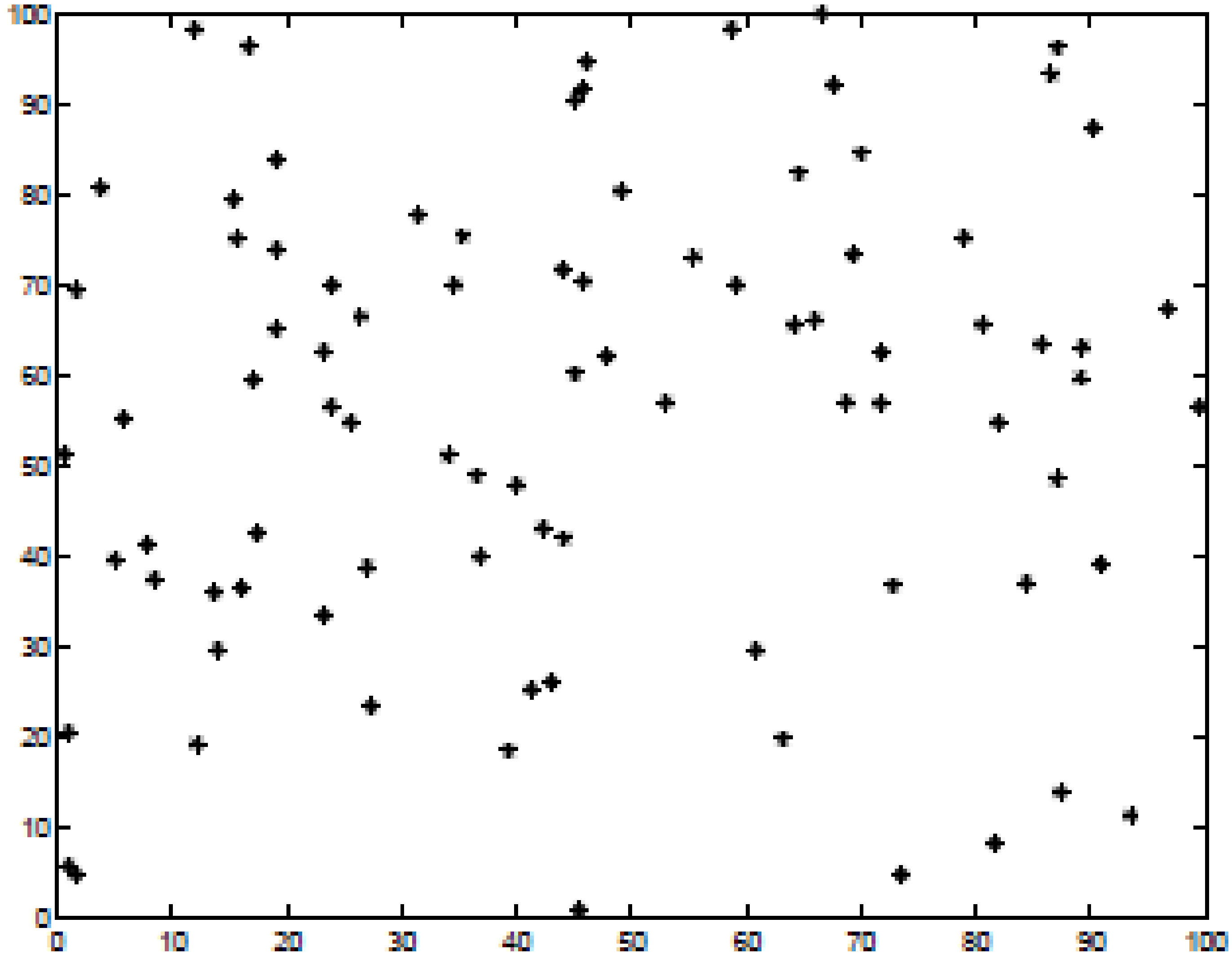
From: Gaetano Kanizsa, Grammatica del Vedere, *Vedere e pensare*, Il Mulino, Bologna



Le « masquage » phénoménologique. Ici juste une différence de précision



Le masquage phénoménologique: ici c’est le nombre d’échantillons du « fond » qui est en cause



Le masquage phénoménologique: ici c’est le nombre d’échantillons du « fond » qui est en cause

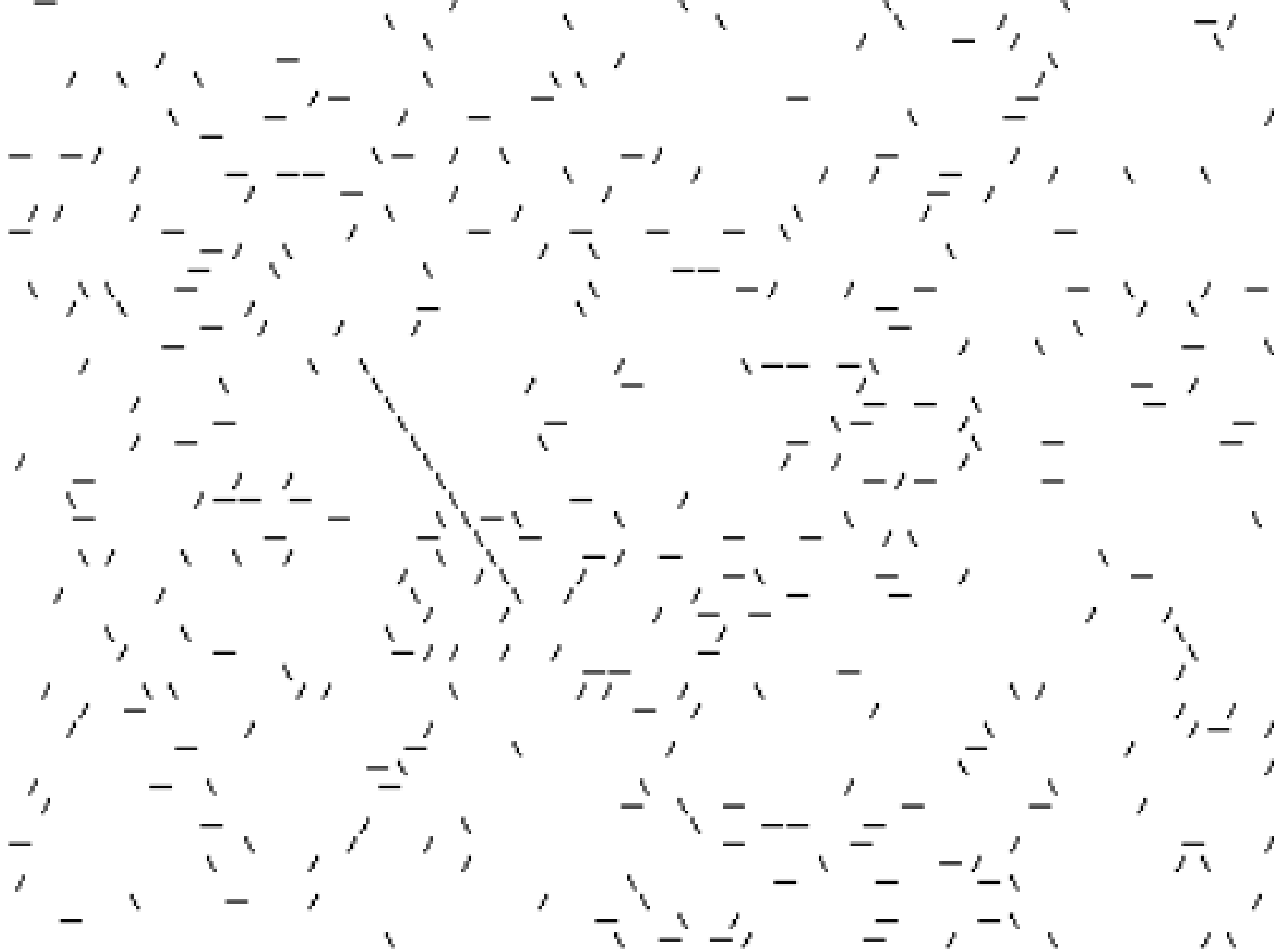


Fig. 13.6. Example of test image used for alignment detection ($N = 50$).

Expérience extraite de Desolneux et al. , From Gestalt theory to image analysis

$$\mathrm{NFA} = \frac{3}{2} N^3 \left(\frac{d}{3}\right)^l .$$

The number $3N^3/2$ approximately counts the number of possible alignments on the image (three orientations, N^2 positions for the center, $N/2$ possibilities for the segment length). The second term is simply the probability that the l cells of the alignment have the proper orientation, knowing the empirical density d of the background small segments. From (13.7), we deduce that the threshold curve in the (d, l) plane corresponding to NFA $\leq \varepsilon$ is defined by

$$l = \frac{C}{\log(d/3)}, \qquad \text{where} \quad C = \log(\varepsilon) - \log\left(\frac{3}{2} N^3\right). \tag{13.8}$$

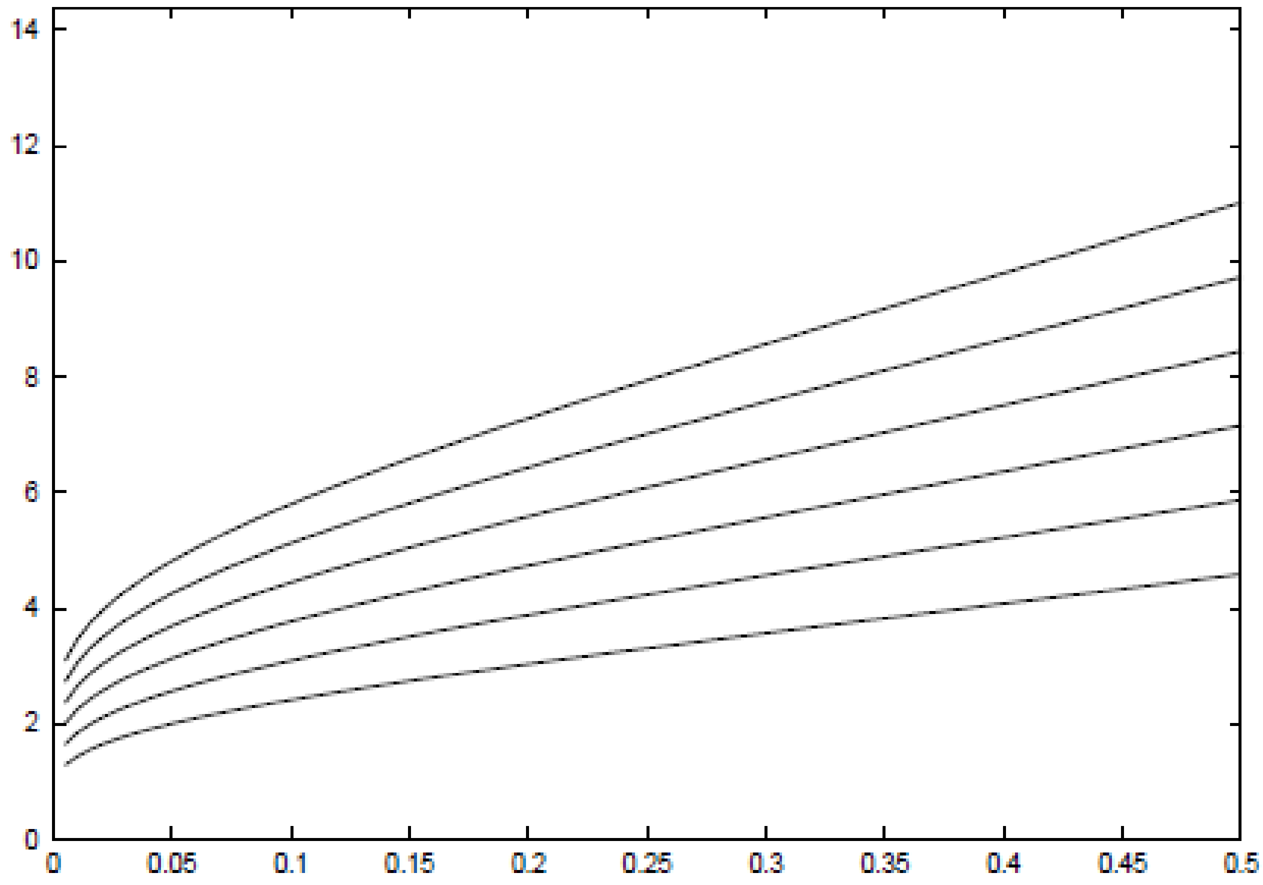


Fig. 13.7. Thresholds in the (d, l) plane (segment density and alignment length) predicted by the Helmholtz principle for different values of ε ($\varepsilon = 10^{-0}, 10^{-1}, 10^{-2}, \ldots, 10^{-5}$) when $N = 50$.

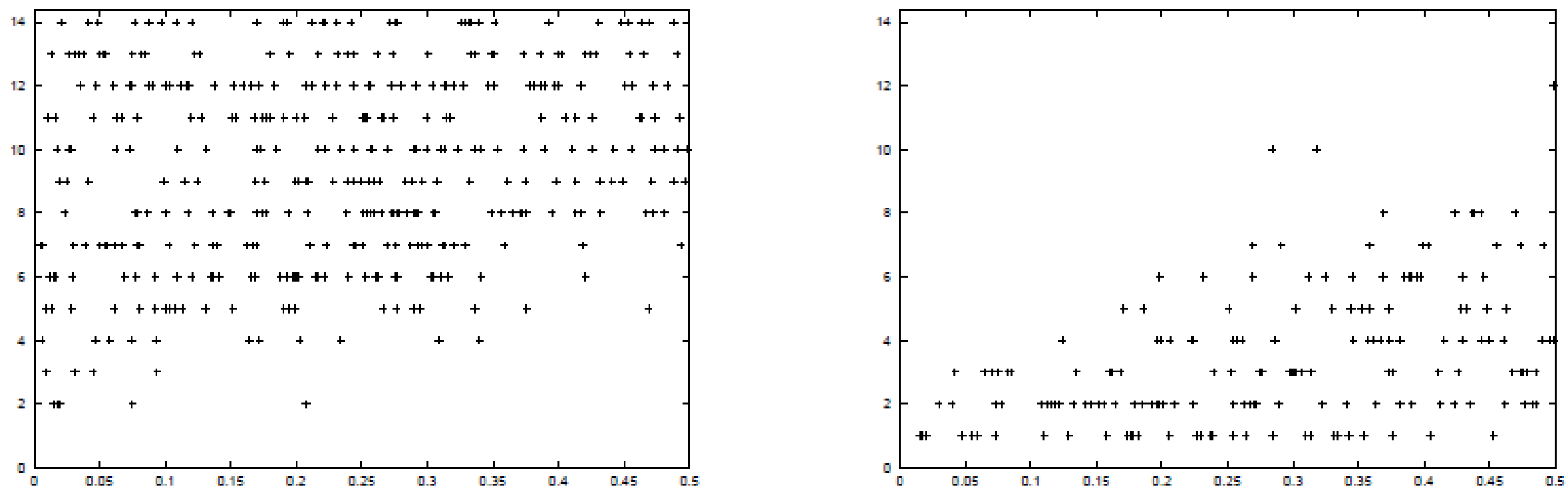


Fig. 13.8. Positive (left) and negative answers (right) in the (d, l) plane (segment density and alignment length).

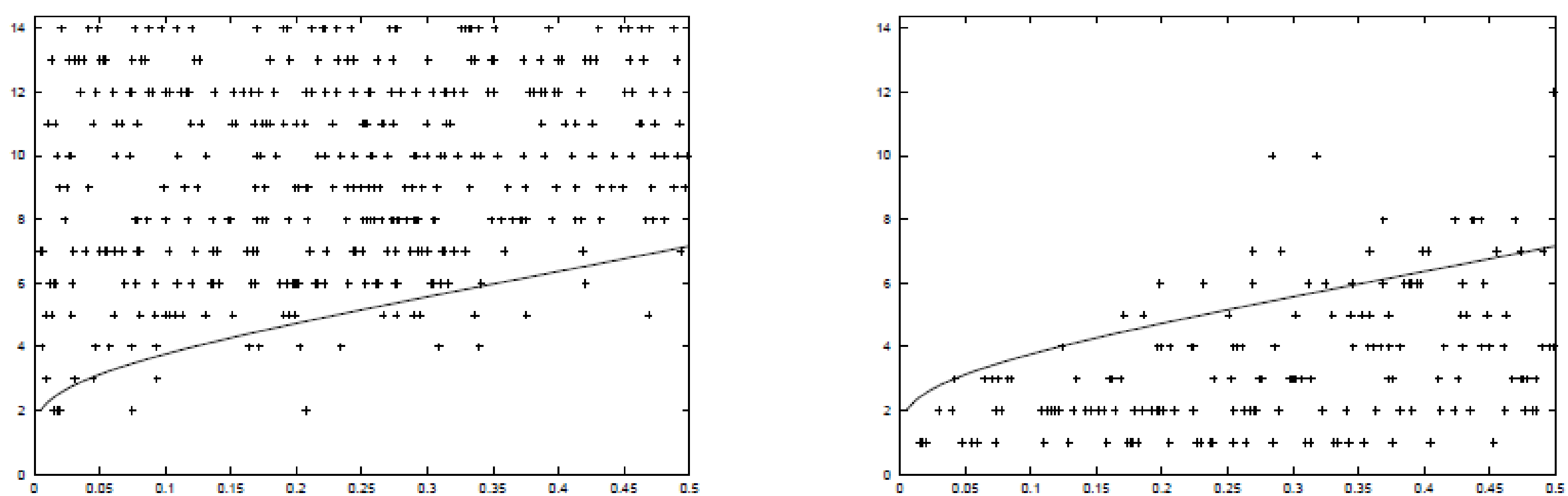


Fig. 13.9. Positive (left) and negative answers (right) and the prediction curve ($\varepsilon = 10^{-2}$).

LSD

- LSD is a Line Segment Detector
- It is based in Burns, Hanson, and Riseman method
- It uses a false detection control based on Desolneux, Moisan, Morel theory.
- LSD is fast, produces precise results, and controls false detections.

Resources

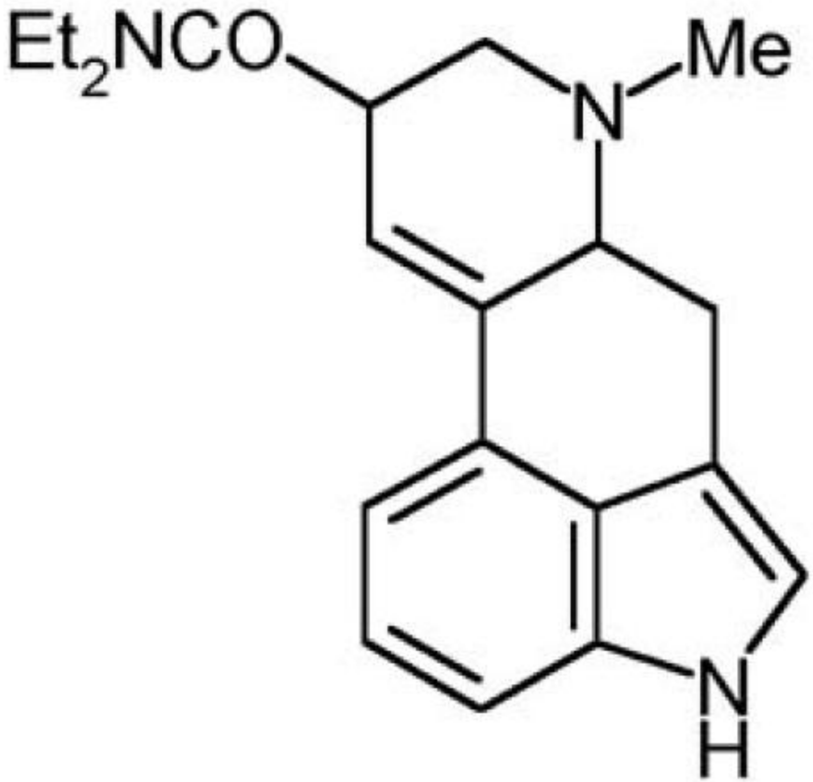
Google: lsd + grompone

Google: lsd + morel

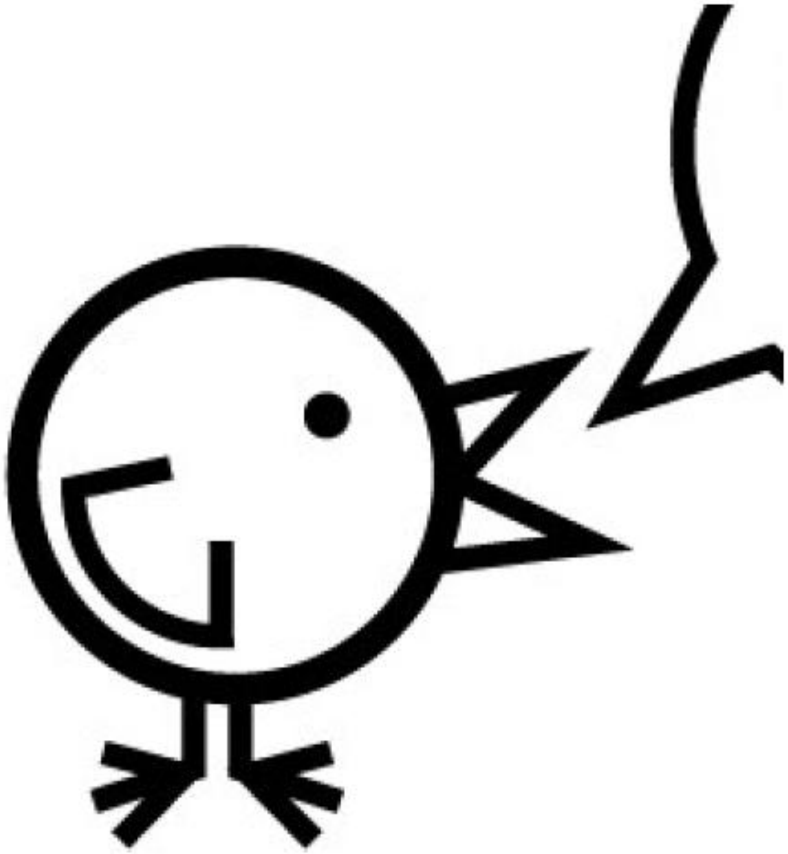
www.ipol.im $\longrightarrow$ LSD: A LINE SEGMENT DETECTOR

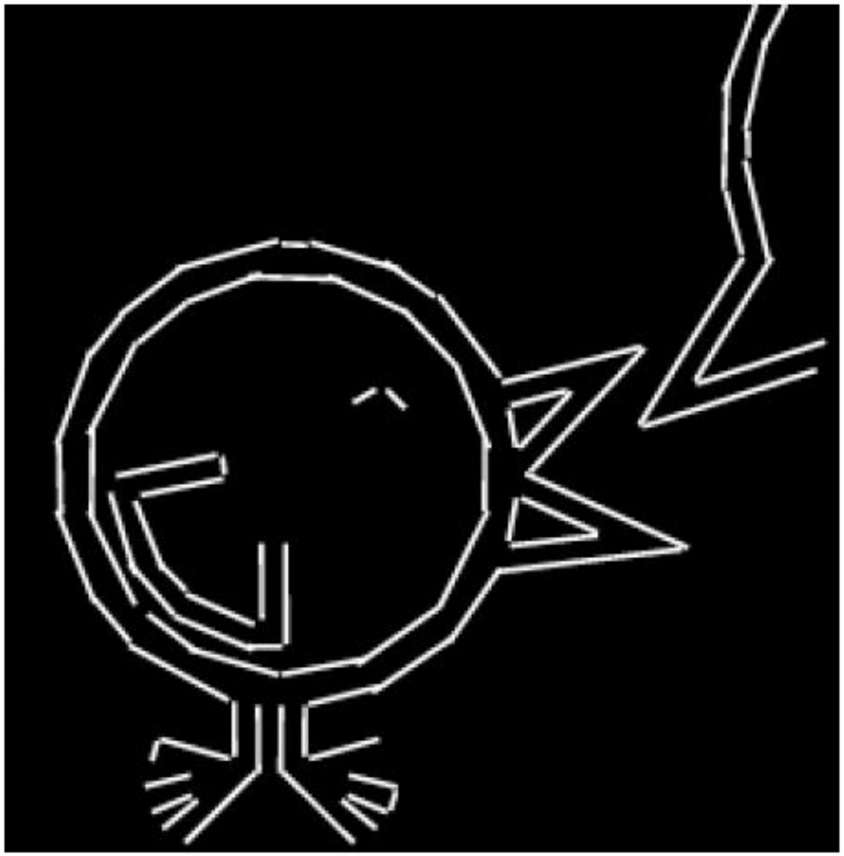
Examples

Examples

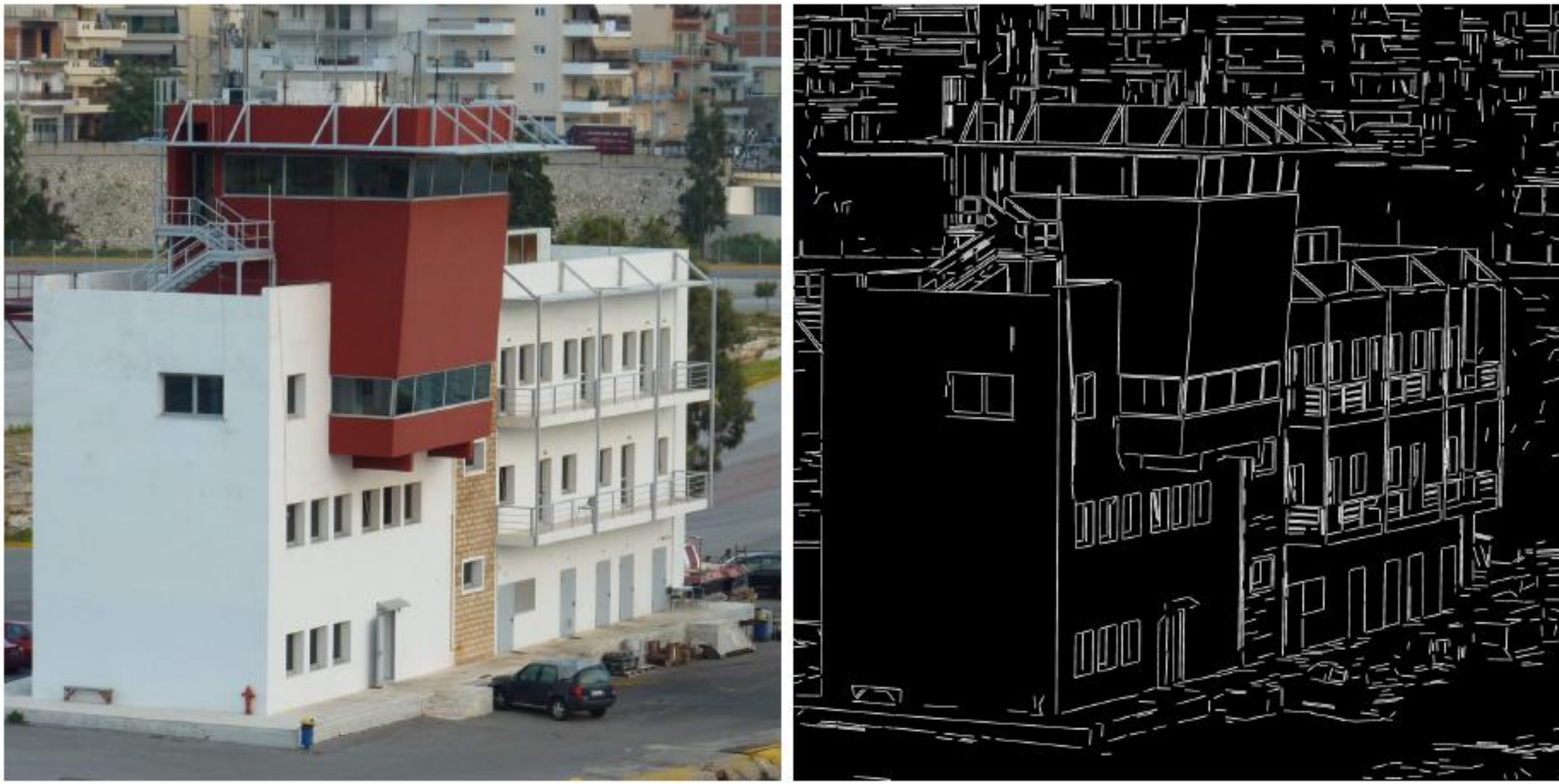


Examples

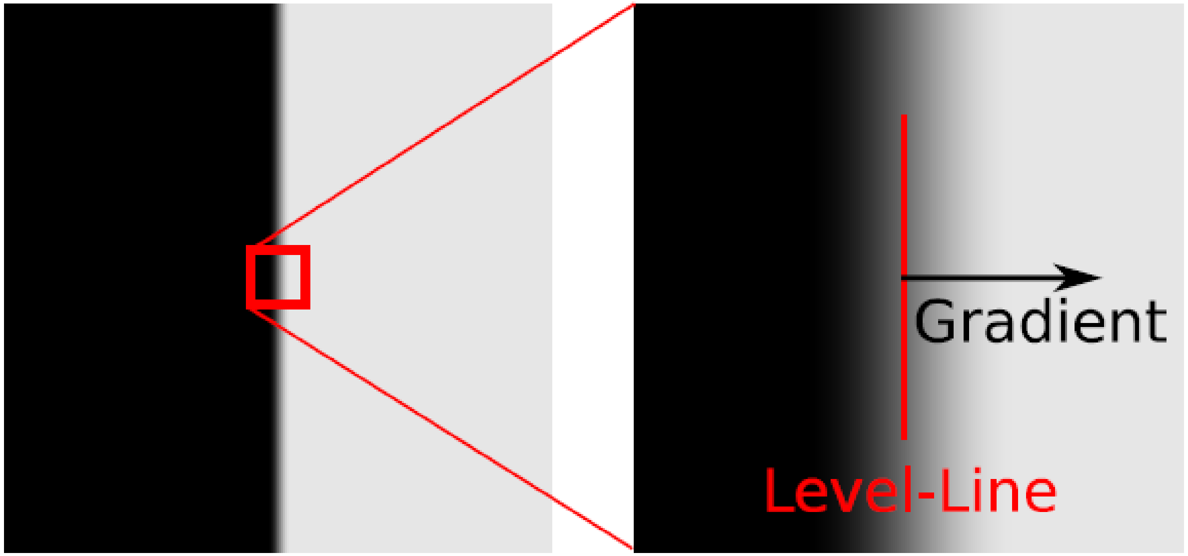


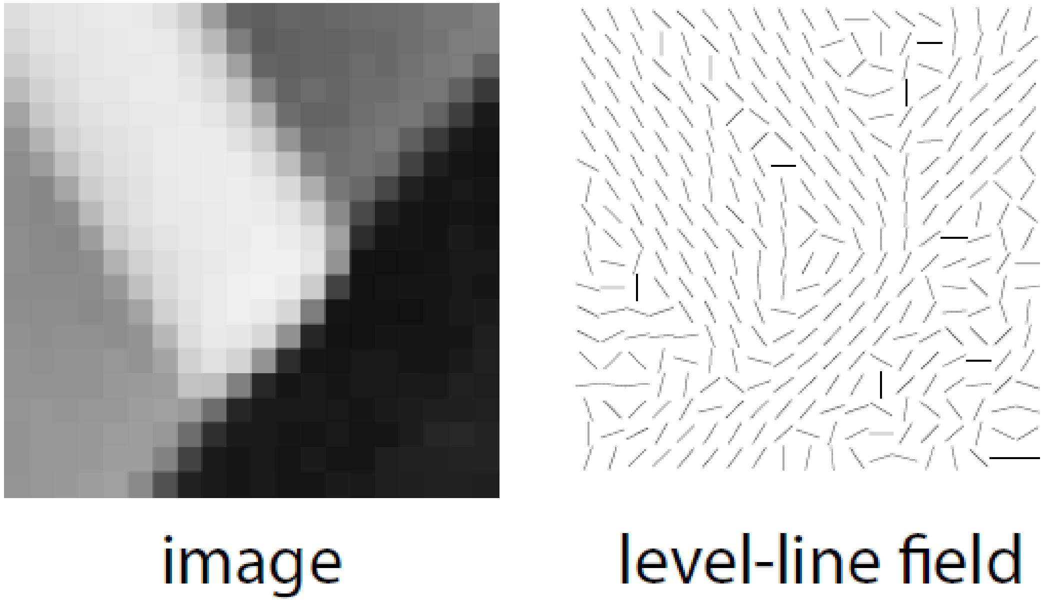


Examples



Gradient and Level-Line Field





LSD in three steps

1. Partition the image into groups of connected pixels that share the same level-line angle up to a certain tolerance
2. Find rectangular approximations
3. Validation

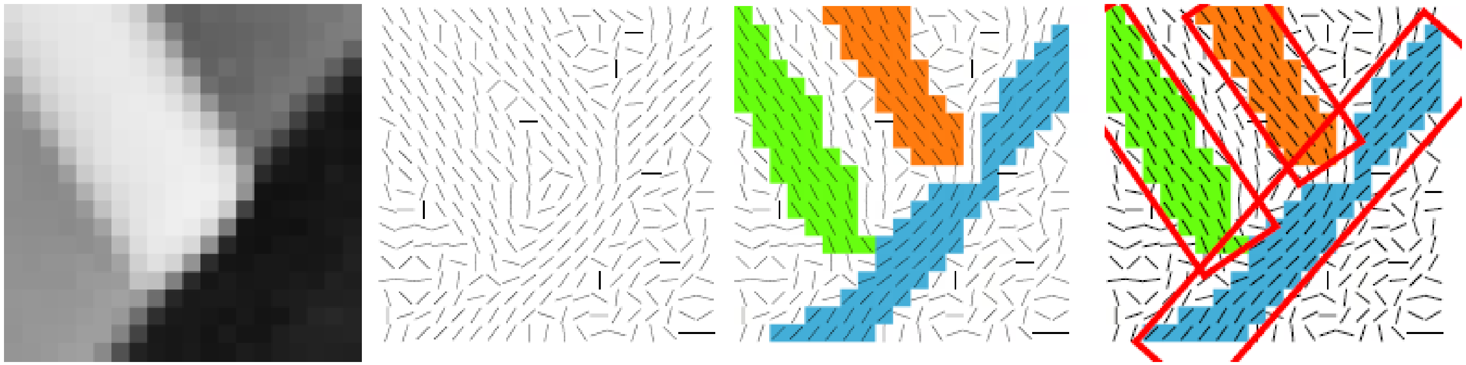
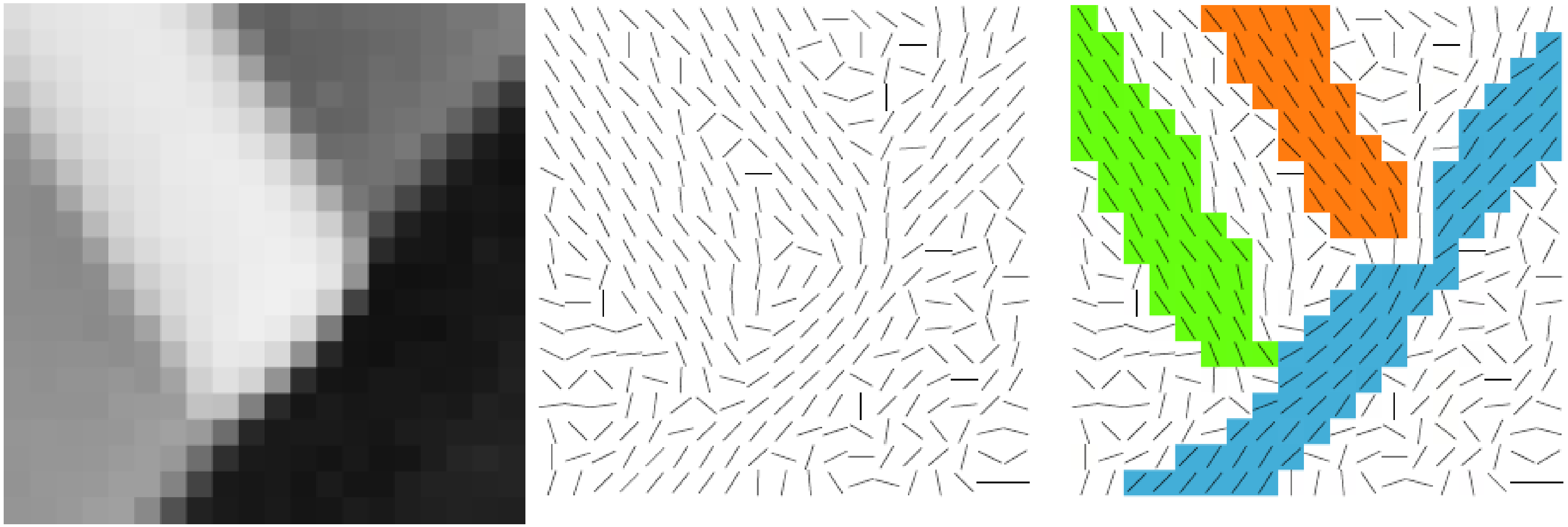
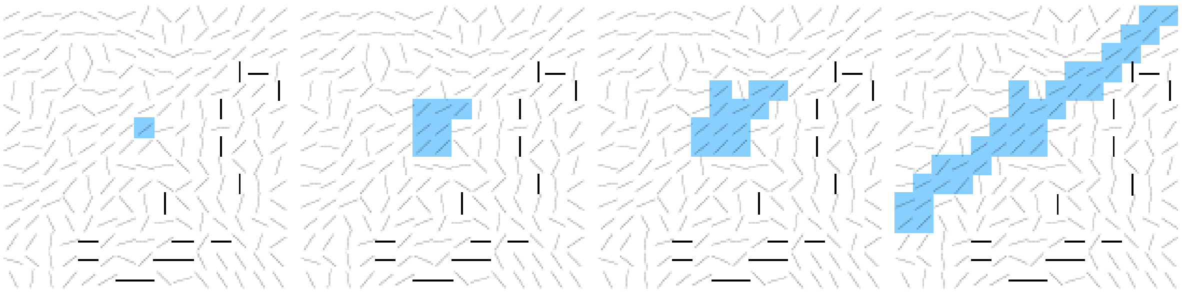


Image Level-line field partition rectangles

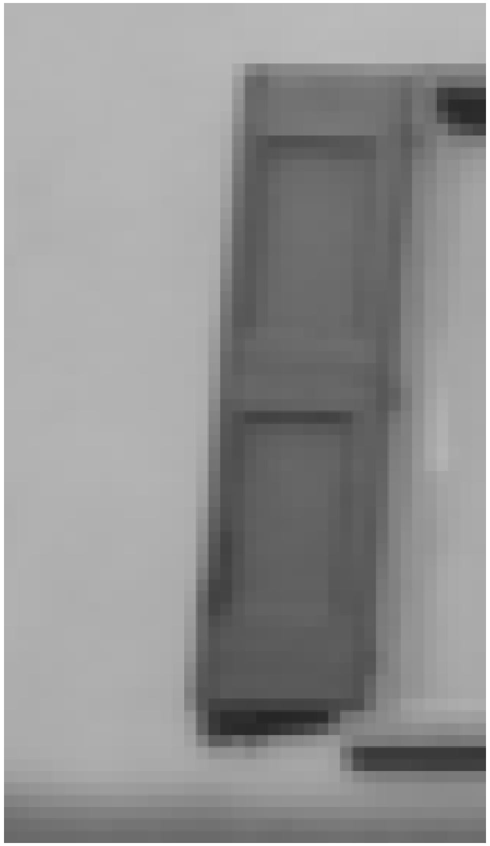
Line-Support Regions

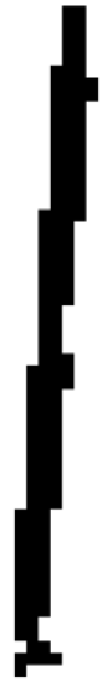
A group of connected pixels that share the same level-line angle up to a certain tolerance.

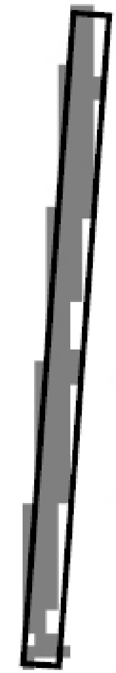




Rectangular Approximation of Regi

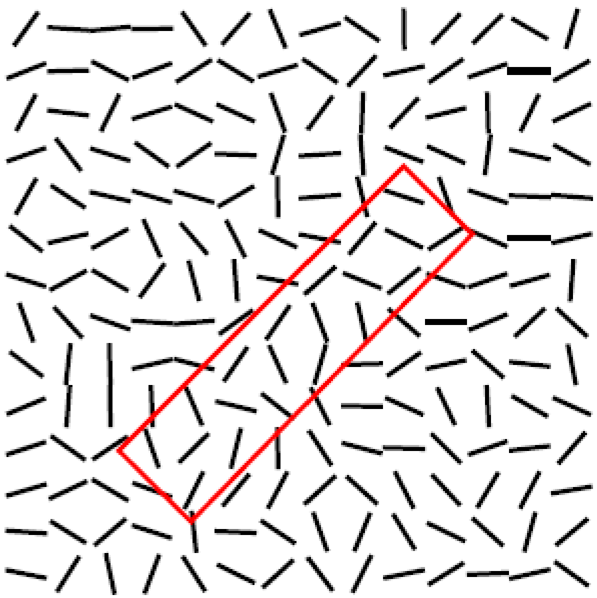




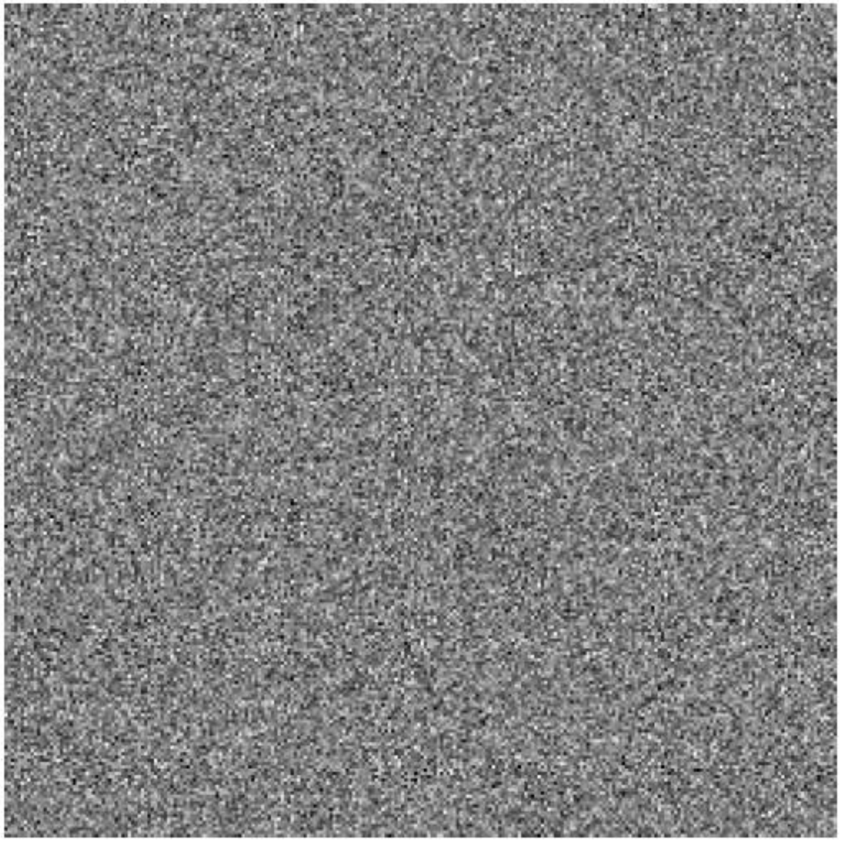


- Pixel's mass is proportional to the gradient modulus
- Region's center of mass $\longrightarrow$ rectangle's center
- First inertia axis of the region $\longrightarrow$ rectangle's angle
- Length and width to envelope most of region's mass

Validation



Helmholtz Principle

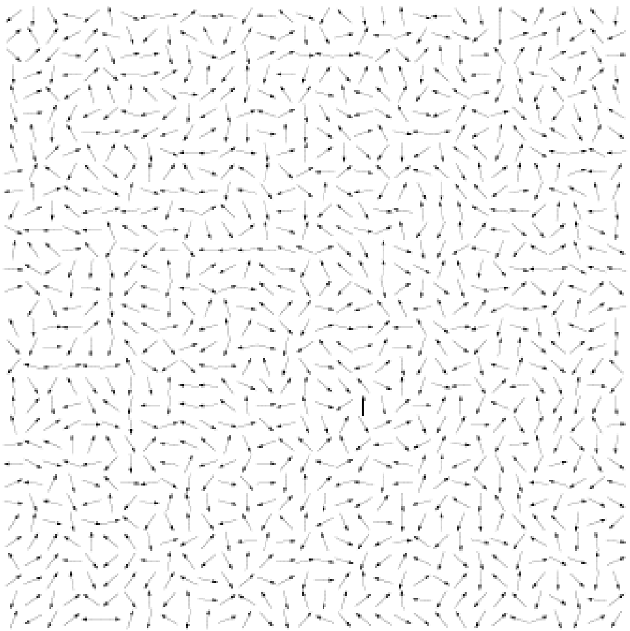


There is no perception on noise.

Fred Attneave (1954) : "Some Informational Aspects of Visual Perception" Psychological Review 61.

A Contrario Detection [Desolneux, Moisan, Morel]

Structure is detected as outliers of a noise model H_0:



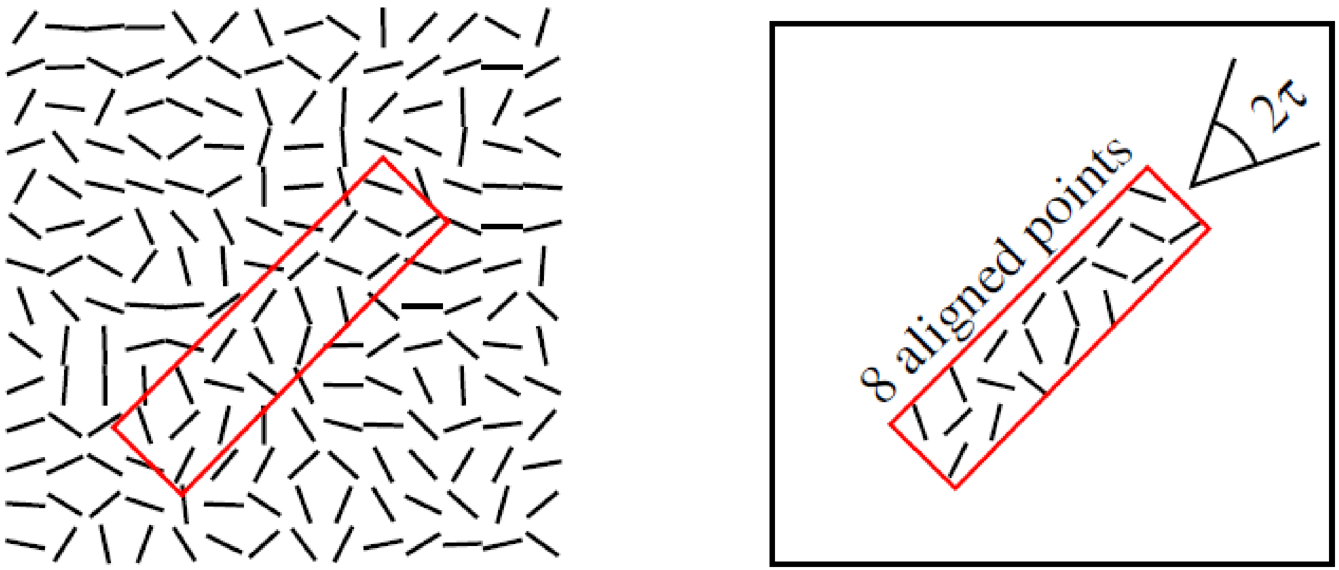
Non-Structured Level-Line Orientations:

- angles are independent random variables
- uniformly distributed in $[0, 2\pi]$

More precisely: an observed geometric structure becomes meaningful when the expectation of its number of occurrences is very small in the non-structured data model.

Aligned Point

A point whose level-line angle is equal to the rectangle angle up to a certain tolerance τ.



$k(r, i)$ is the number of aligned points of rectangle r in image i.
$n(r)$ is the total number of pixels in the rectangle r.
In the example, $k(r, i) = 8$ and $n(r) = 27$.

Meaningful Rectangle

Given a rectangle r with $k(r,i)$ observed aligned points, we define

$$\mathrm{NFA}(r,i) = N_{test} \cdot P_{H_0}\left[k(r,I) \geq k(r,i)\right]$$

where:

I is a random image on H_0,

N_{test} is the number of tests.

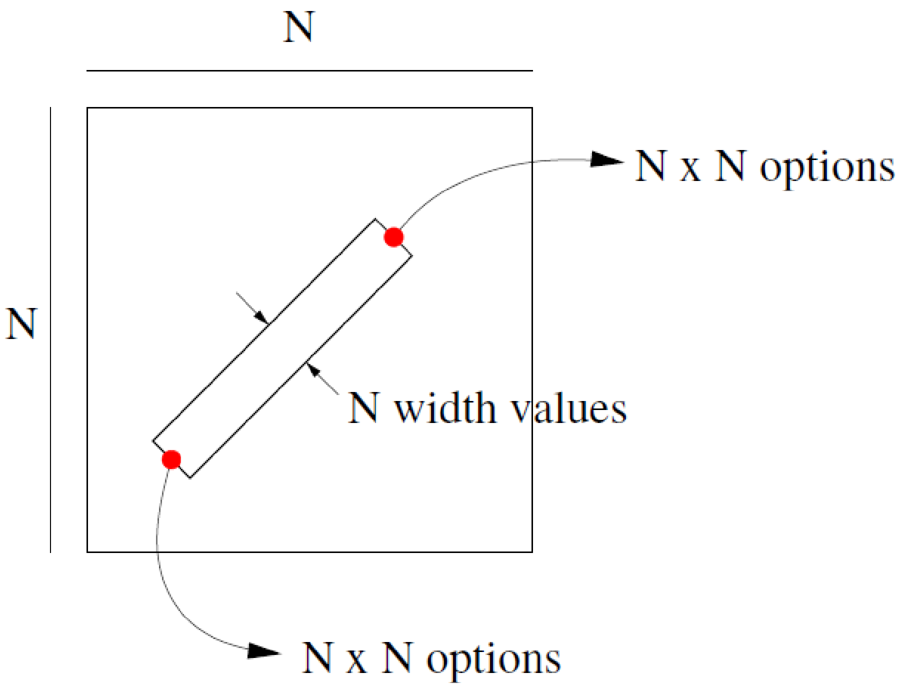
$\mathrm{NFA}(r,i)$ is the expected number of event as good as (r,i) in H_0.

When $\mathrm{NFA}(r,i)$ is large: a common event in H_0 and not meaningful.

When $\mathrm{NFA}(r,i)$ is small: a rare event in H_0 and probably meaningful.

A rectangle with $\mathrm{NFA}(r,i) \leq \varepsilon$ is called ε-meaningful rectangle.

Number of Tests



$$N_{test} = N^5$$

Probability term

In H_0, the probability that a pixel is an aligned point is

$$p = \frac{\tau}{\pi}.$$

Because of the independence in H_0, $k(r, l)$ follows a binomial distribution. Then,

$$P_{H_0}\left[k(r, l) \geq k(r, i)\right] = B\big(n(r), k(r, i), p\big)$$

where $B(n, k, p)$ is the tail of the binomial distribution:

$$B(n, k, p) = \sum_{j=k}^{n} \binom{n}{j} p^j (1 - p)^{n-j}$$

NFA

The final expression for the Number of False Alarms for a rectangle is:

$$\mathrm{NFA}(r, i) = N^5 \cdot \sum_{j=k(r,i)}^{n(r)} \binom{n(r)}{j} p^j (1-p)^{n(r)-j}$$

Theorem

$$E_{H_0}\left[\sum_{r\in\mathcal{R}} \mathbb{1}_{\mathrm{NFA}(r,I)\le\varepsilon}\right] \le \varepsilon$$

where E is the expectation operator, $\mathbb{1}$ is the indicator function, $\mathcal{R}$ is the set of rectangles considered, and I is a random image in H_0.

The theorem states that the average number of ε-meaningful rectangles on the a contrario model H_0 images is less than ε.

In other words, it shows that LSD satisfies the Helmholtz principle.

Proof

We define $\hat{k}(r)$ as

$$\hat{k}(r) = \min\left\{n \in \mathbb{N},\ P_{H_0}\left[k(r,l) \geq n\right] \leq \frac{\varepsilon}{N^5}\right\}.$$

Then, $\mathrm{NFA}(r,i) \leq \varepsilon$ is equivalent to $k(r,i) \geq \hat{k}(r)$. Now,

$$E_{H_0}\left[\sum_{r\in\mathcal{R}} \mathbb{1}_{\mathrm{NFA}(r,l)\leq\varepsilon}\right] = \sum_{r\in\mathcal{R}} P_{H_0}\left[\mathrm{NFA}(r,l) \leq \varepsilon\right] = \sum_{r\in\mathcal{R}} P_{H_0}\left[k(r,l) \geq \hat{k}(r)\right].$$

But, by definition of $\hat{k}(r)$ we know that

$$P_{H_0}\left[k(r,l) \geq \hat{k}(r)\right] \leq \frac{\varepsilon}{N^5}$$

and using that $\#\mathcal{R} = N^5$ we get

$$E_{H_0}\left[\sum_{r\in\mathcal{R}} \mathbb{1}_{\mathrm{NFA}(r,l)\leq\varepsilon}\right] \leq \sum_{r\in\mathcal{R}} \frac{\varepsilon}{N^5} = \varepsilon.$$

$\varepsilon = 1$

The result is not very sensible to the value of ε.

image

$\varepsilon = 1$

$\varepsilon = 0.1$

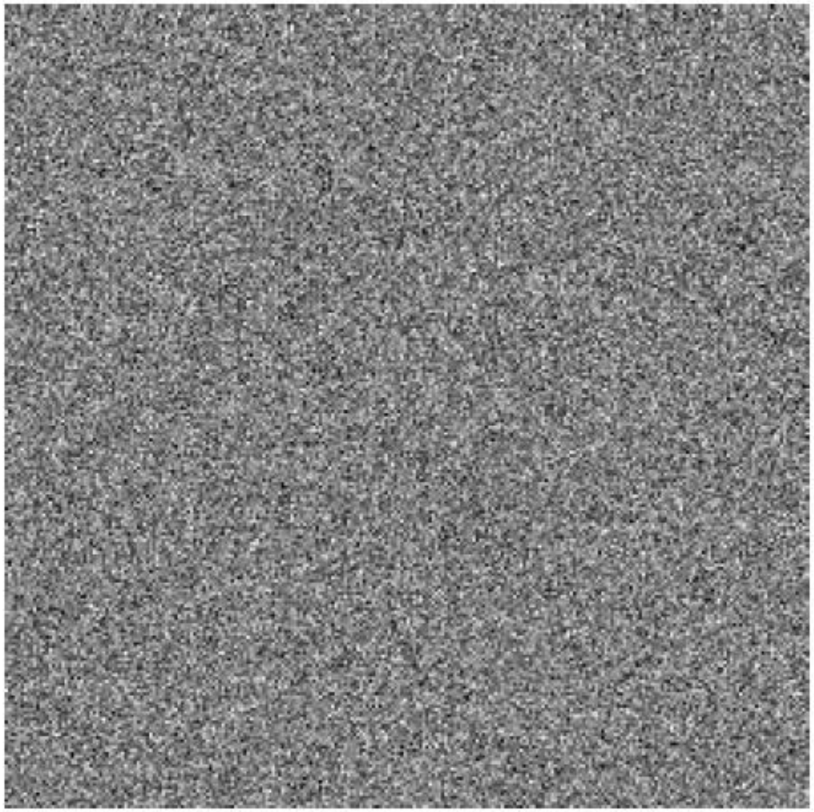
$\varepsilon = 0.01$

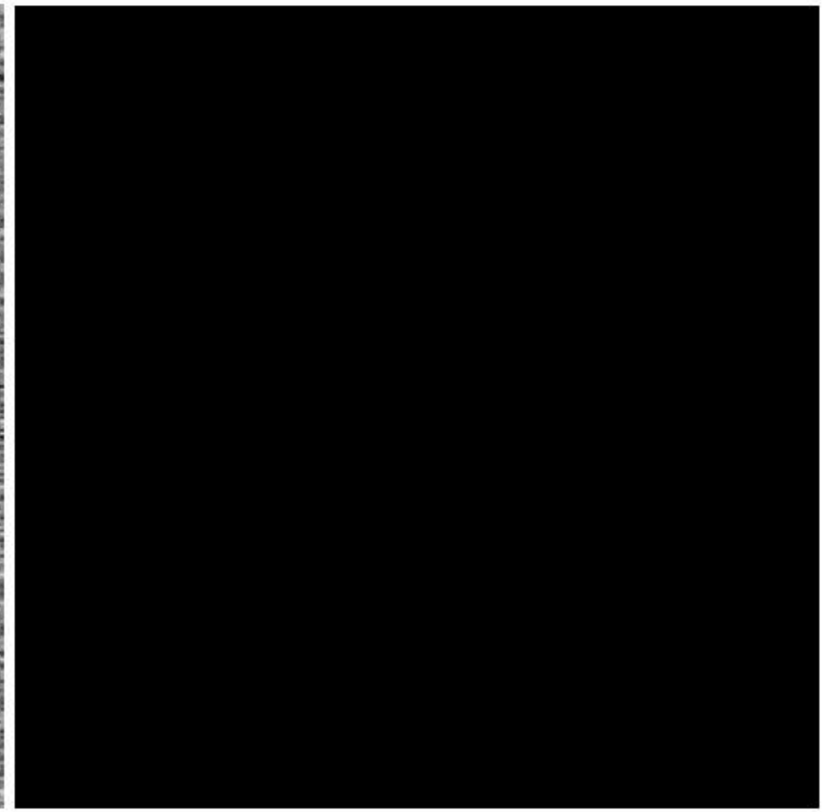
$\varepsilon = 1$ means, on average, one false detection per image.

Algorithm Summary

1. Partition the image into Line-Support Regions
2. For each Line-Support Region:
3. Find the Rectangular Approximation
4. Compute NFA value
5. Rectangles with $\mathrm{NFA} \leq 1$ are added to the output.

Examples





Examples

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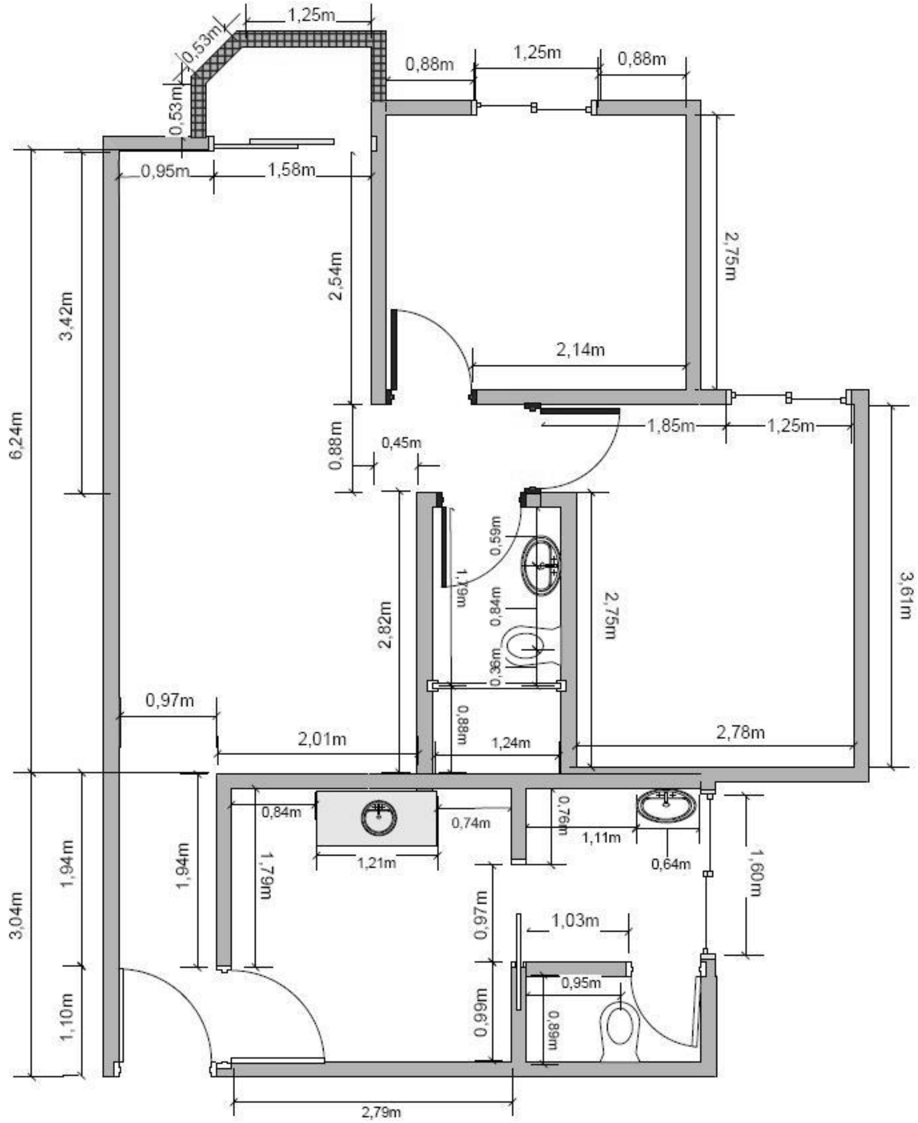
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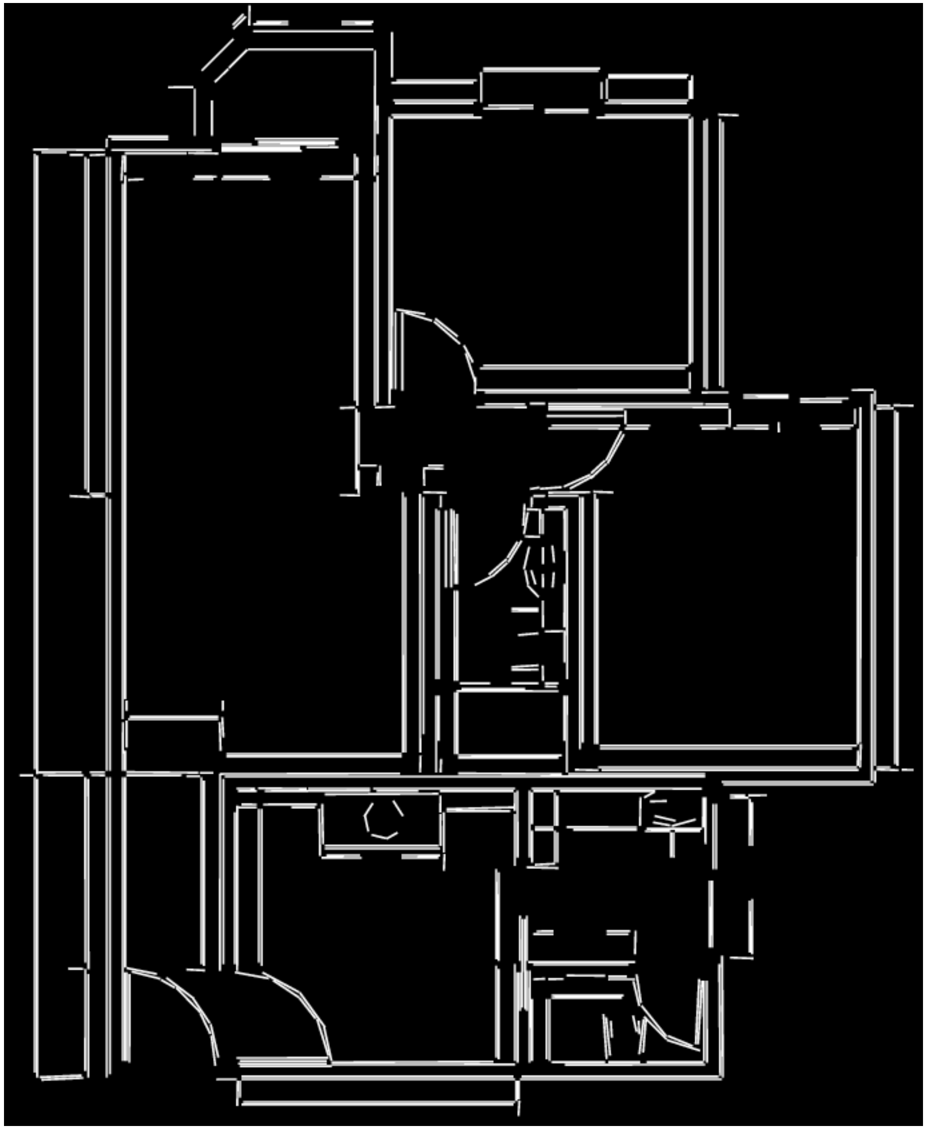

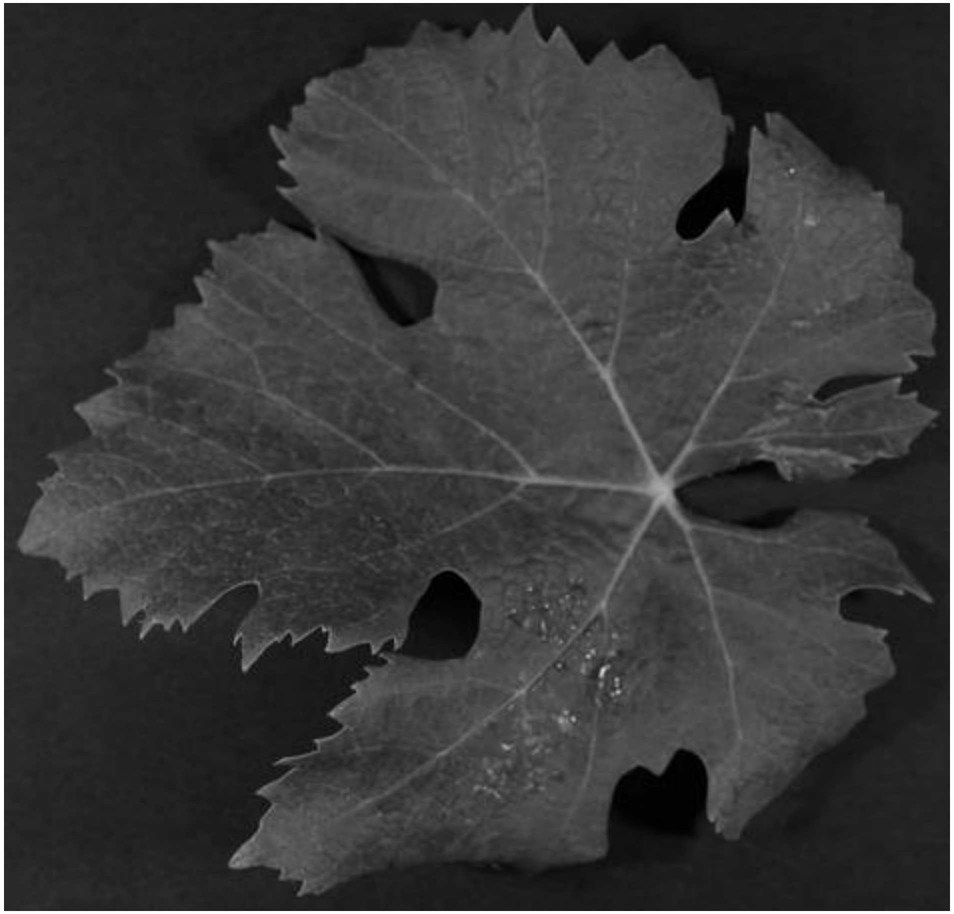

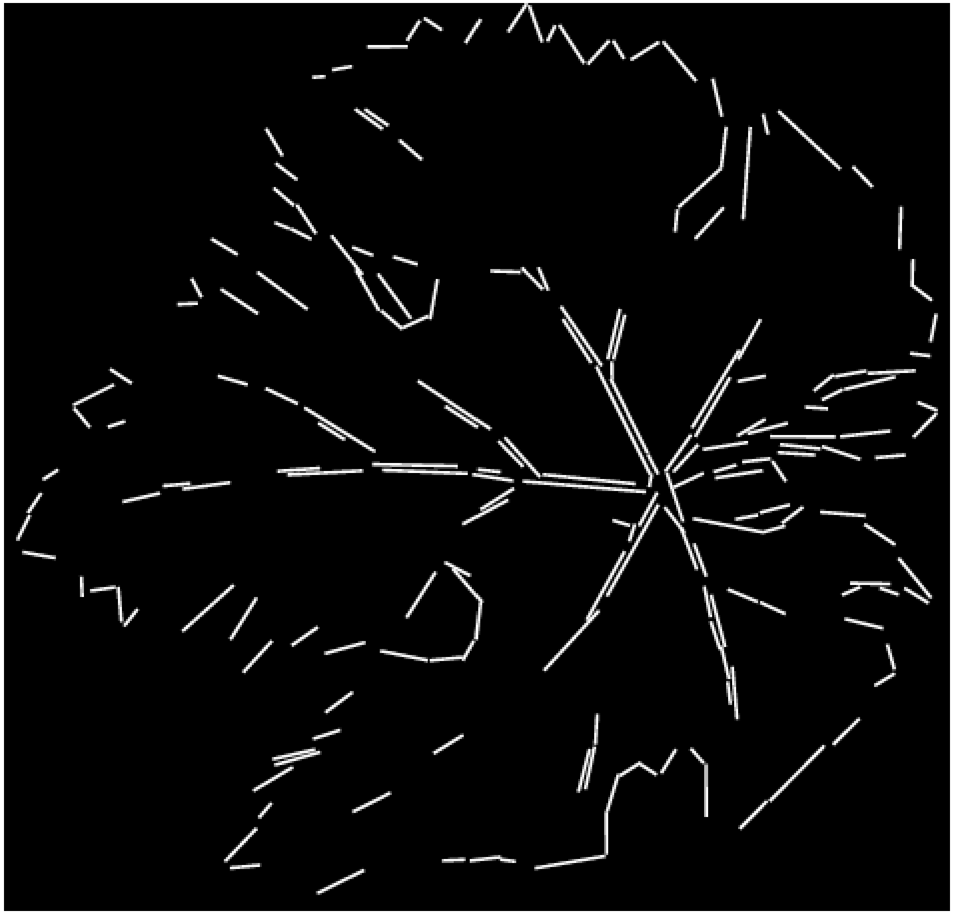

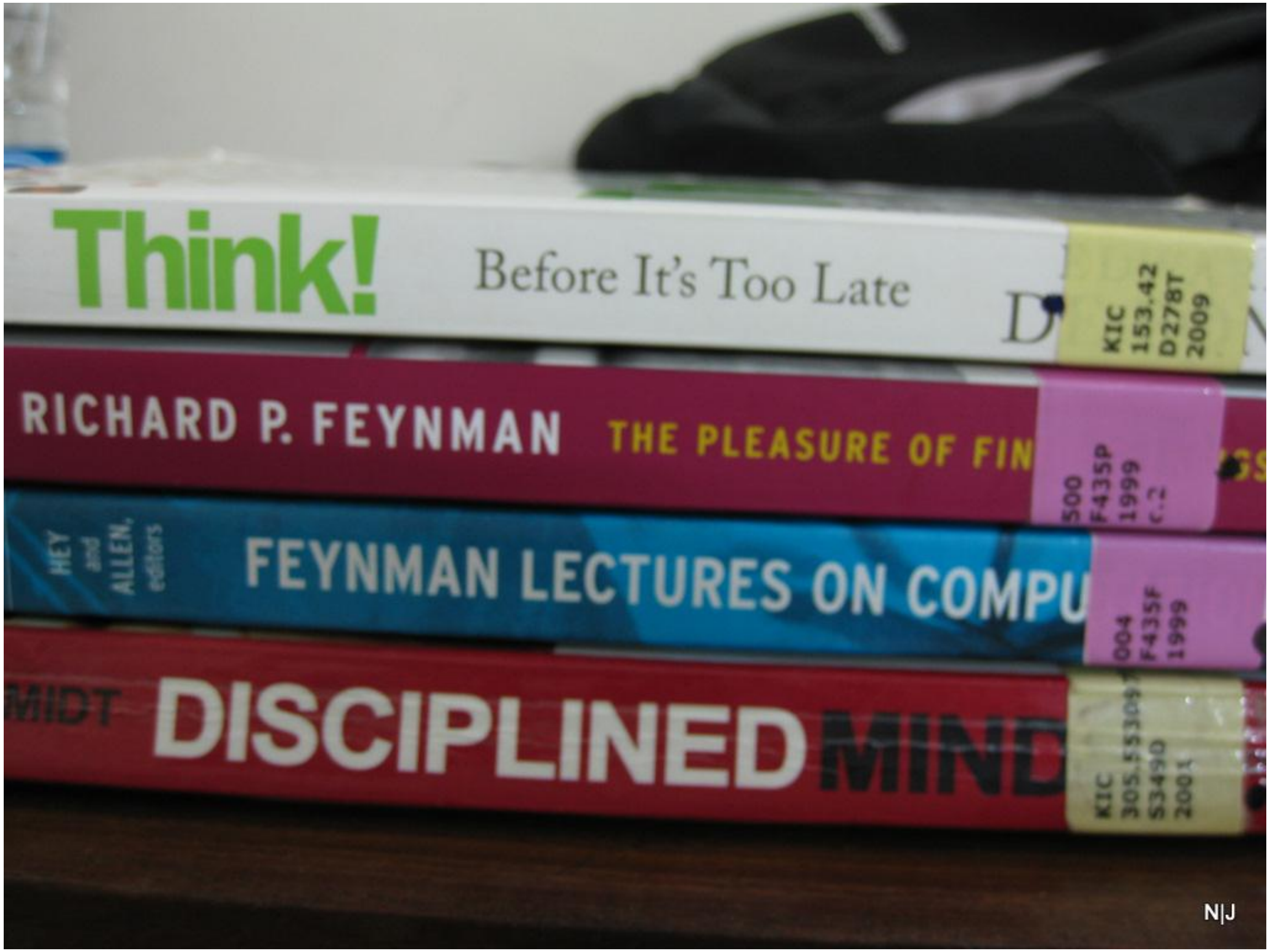
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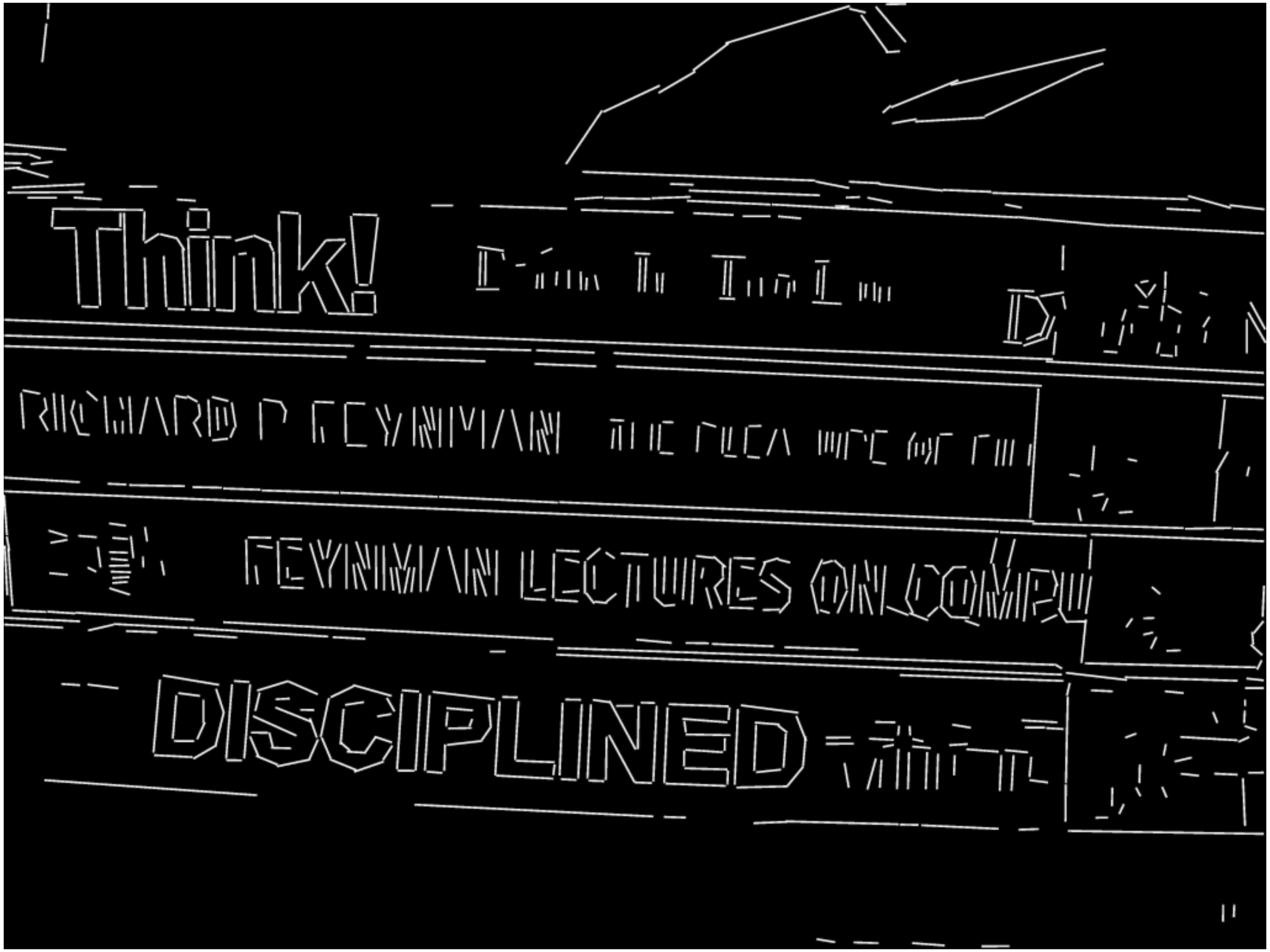
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通信地址						收信人		邮编	
联系电话	移动：		宅电：				入社日期		

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教育背景（请从初中阶段开始填写）	期 间	学校名称	专 业	学 历	学 位	学校所在地
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	主要课程（按最高学历）			GPA及排名（请在申请书后附成绩单）		

英 语 水 平	计算机水平		其它外语及水平（如韩国语等）	
英语等级考试：四级□ 六级□ 专业四级□ 专业八级□	证书：			
TOFEL：未参加□ 参加______分 GRE： 未参加□ 参加______分 GMAT： 未参加□ 参加______分	常用办公软件名称	水平	预计到岗时间：	
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口语：一般□ 较好□ 优秀□ 其他证书：	IT项目经验：		期望月薪（税前）：RMB	

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Algorithm 1 (Desolneux et al. : détecter des alignements de points)

Thus, if the probability of having one point inside the strip is p, and we observe k points inside the strip (without counting the two points defining it) out of N points, the NFA value is:

$$NFA = N_{tests} \times \mathcal{B}(k, N, p) \qquad (1)$$

The number of tests, N_{tests} is the number of all possible couple of points times the number of the considered possible strip widths. If N is the number of points in the image, and Ω the set of considered widths.

$$N_{tests} = \frac{N \times (N-1)}{2} \times |\Omega| \qquad (2)$$

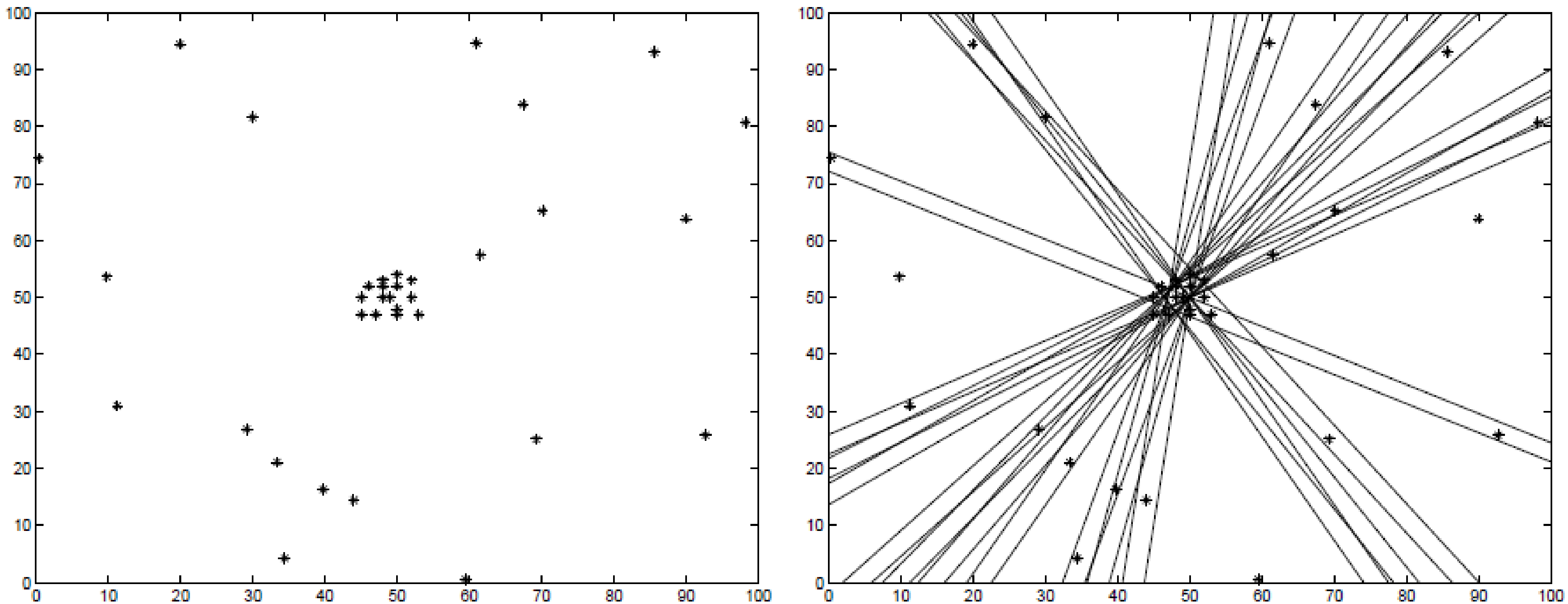


Fig. 14.6. One cluster or several alignments?

3 Algorithm 2

3.1 Description

Algorithm 2 also goes through each possible couple of points, and through each possible precision value and decides if the segment formed by two points is meaningful or not, based on the amount of points lying between them, at the given precision.

Contrary to algorithm 1, in this case we will deal with the situation of having clusters of points, and we will make the estimation of the points denisity locally. This two problems and our proposed solution are explained in the following subsections.

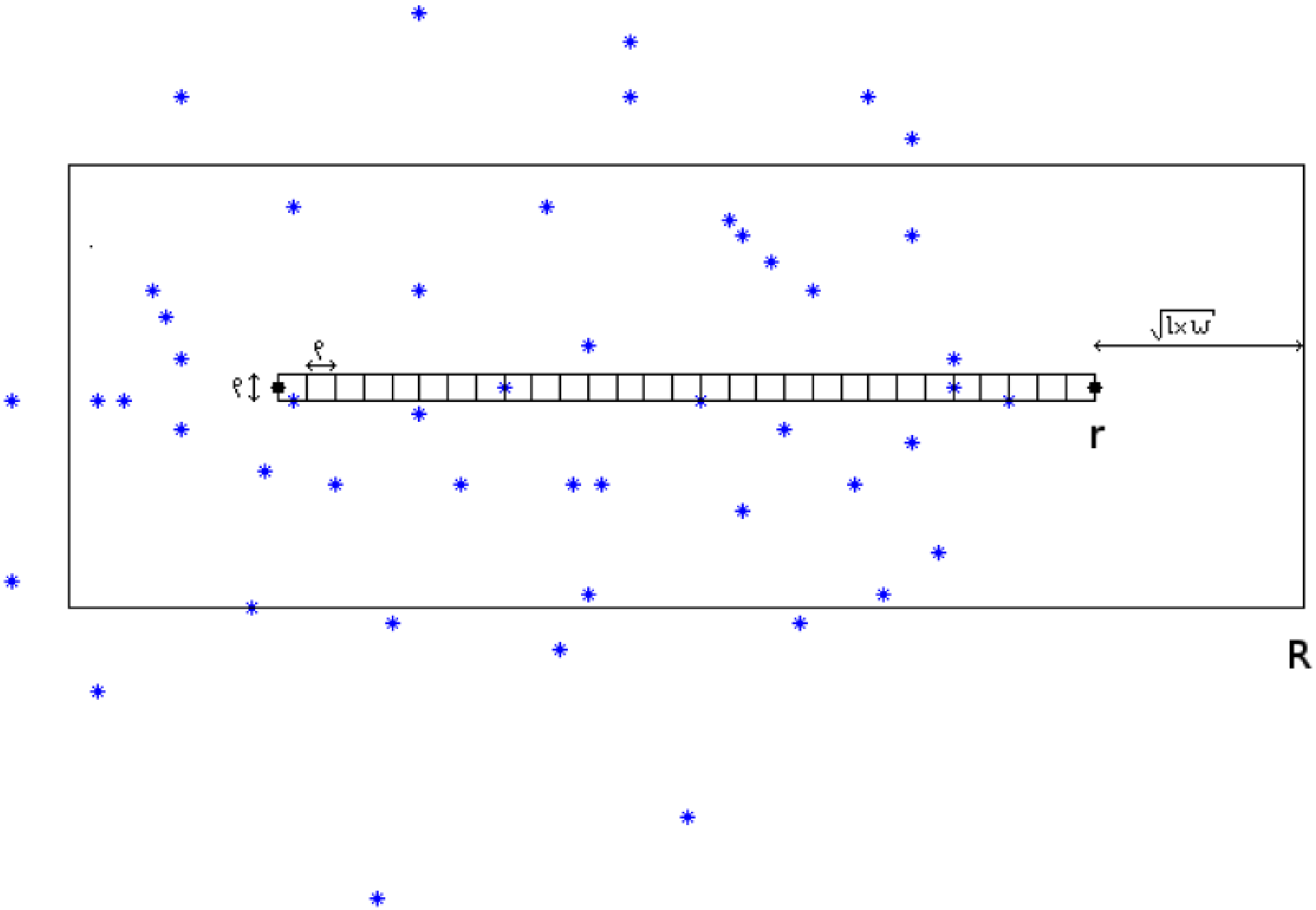


Figure 1: Two points and a precision ρ define a strip r. The strip is divided in square cases, and the density of points is estimated from the surrounding rectangular R

In our uniform model, the probability of having one of the M points inside the square is the ratio of areas between the small square area and the area of the rectangle R, $p_1 = \frac{|r|}{|R|}$. Then,

$$p_2 = B(1, M, p_1)$$

$$p_3 = B(k, n, p_2)$$

Finally, the NFA for this event is

$$NFA = N_{test} \times p_3$$

The number of tests is defined in the same way as for algorithm 1.

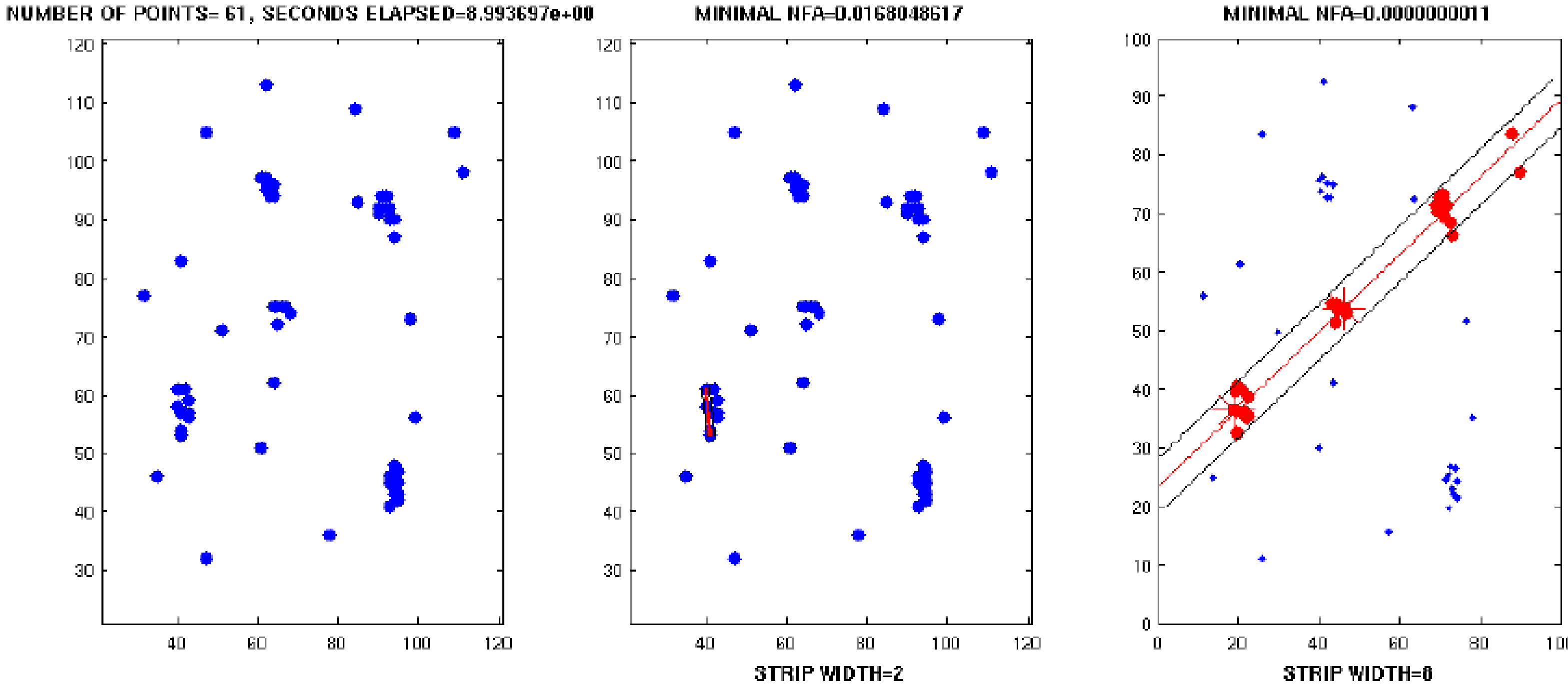


Figure 5: From left to right: original image, result of algorithm 2, result of algorithm 1 In this case we observe the effect of point clusters in the result of algorithm 1. For algorithm 1, all the points in the obtained strip are counted and thus the strip appears as highly meaningful. The effect of using the square cases in algorithm 2 is that the clusters will be counted just once and thus the meaningfulness of that strip is reduced.

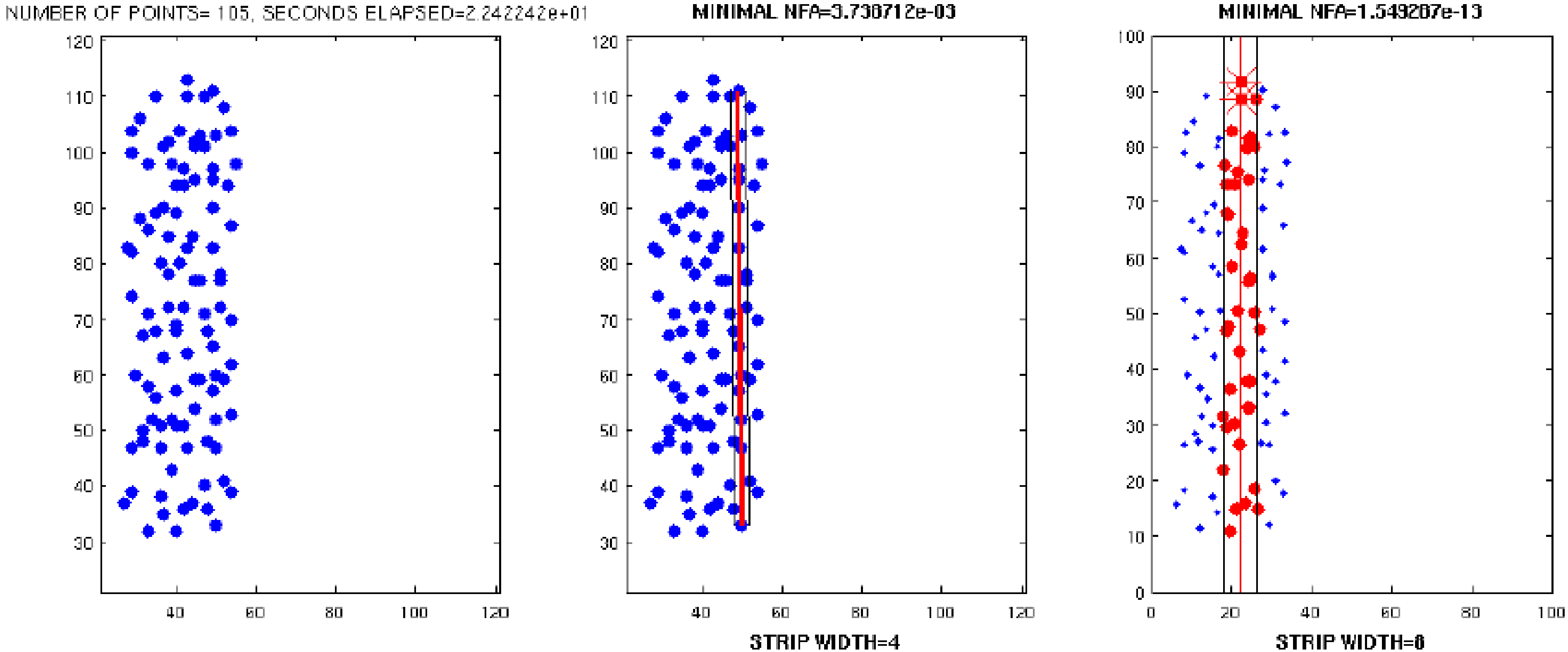


Figure 6: From left to right: original image, result of algorithm 2, result of algorithm 1. In this case we see the effect of estimating the density of points from a local region around the considered strip or from the whole image. In the case of algorithm 1, the density is estimated on the whole image. Under the assumption of points uniformly scattered in the image, the concentration of points in the obtained strip is very meaningful. In the case of algorithm 2, the density is estimated in an area surrounding the segment and thus it has lower meaningfulness

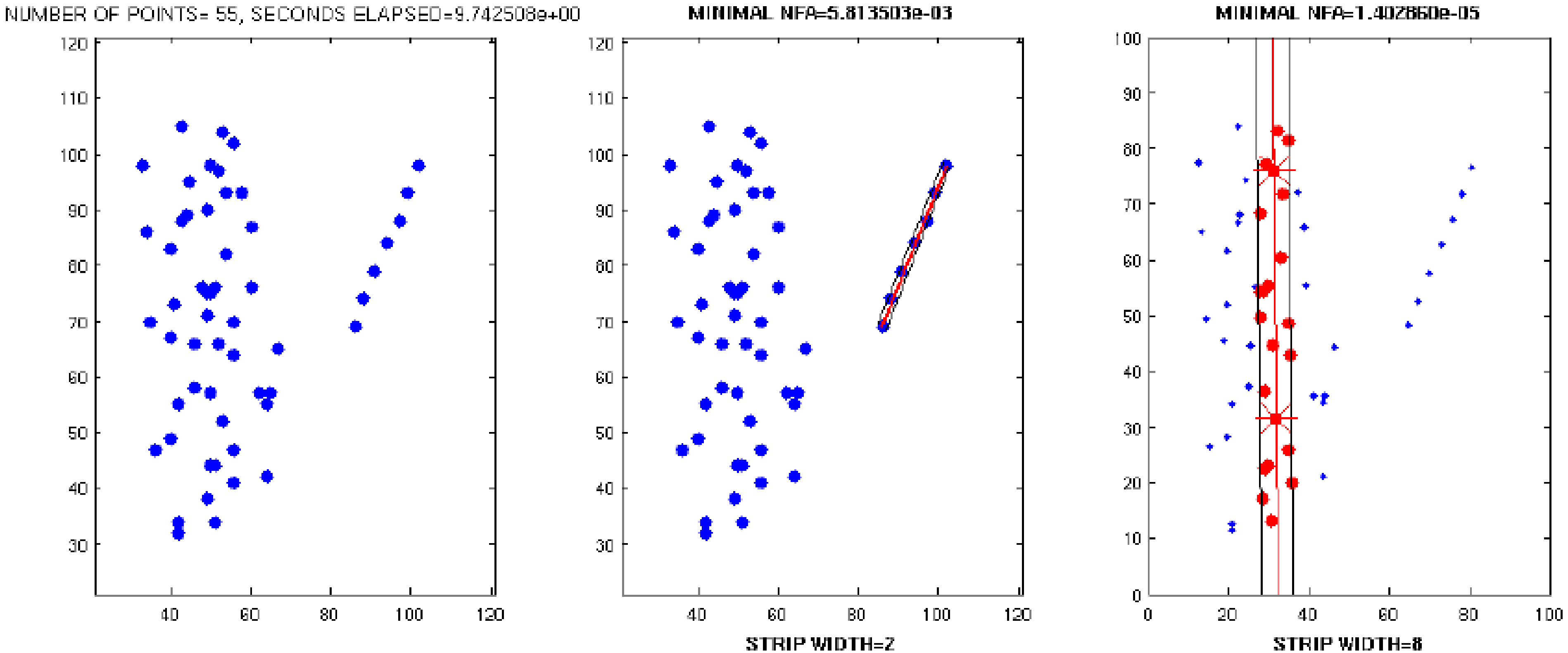


Figure 7: From left to right: original image, result of algorithm 2, result of algorithm 1. In this case we observe again the effect of estimating the density of points locally. Here, algorithm 2 is able to correctly identify the perceptually meaningful alignemnt.

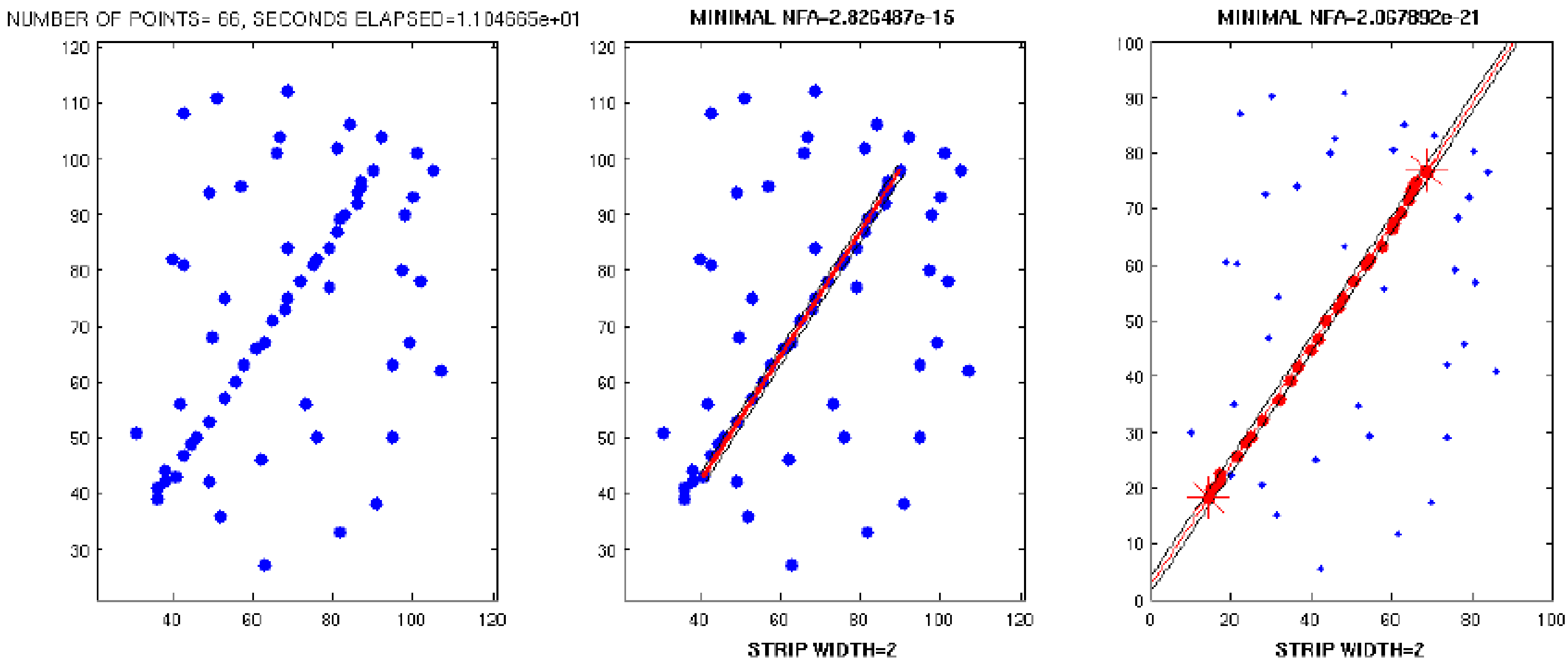


Figure 8: From left to right: original image, result of algorithm 2, result of algorithm 1. Here we observe again the effect of point clusters. Notice how the result from algorithm 2 is less meaninfgul than the one from algorithm 1

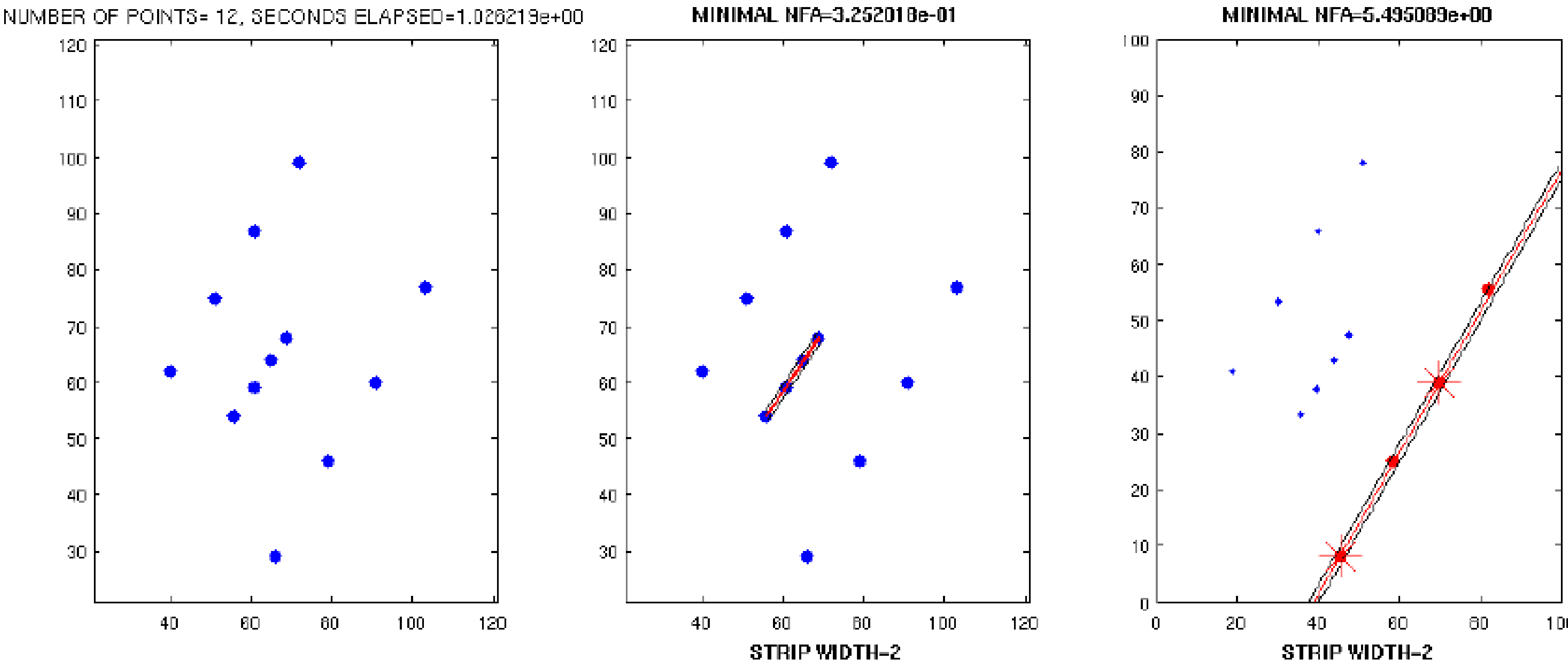
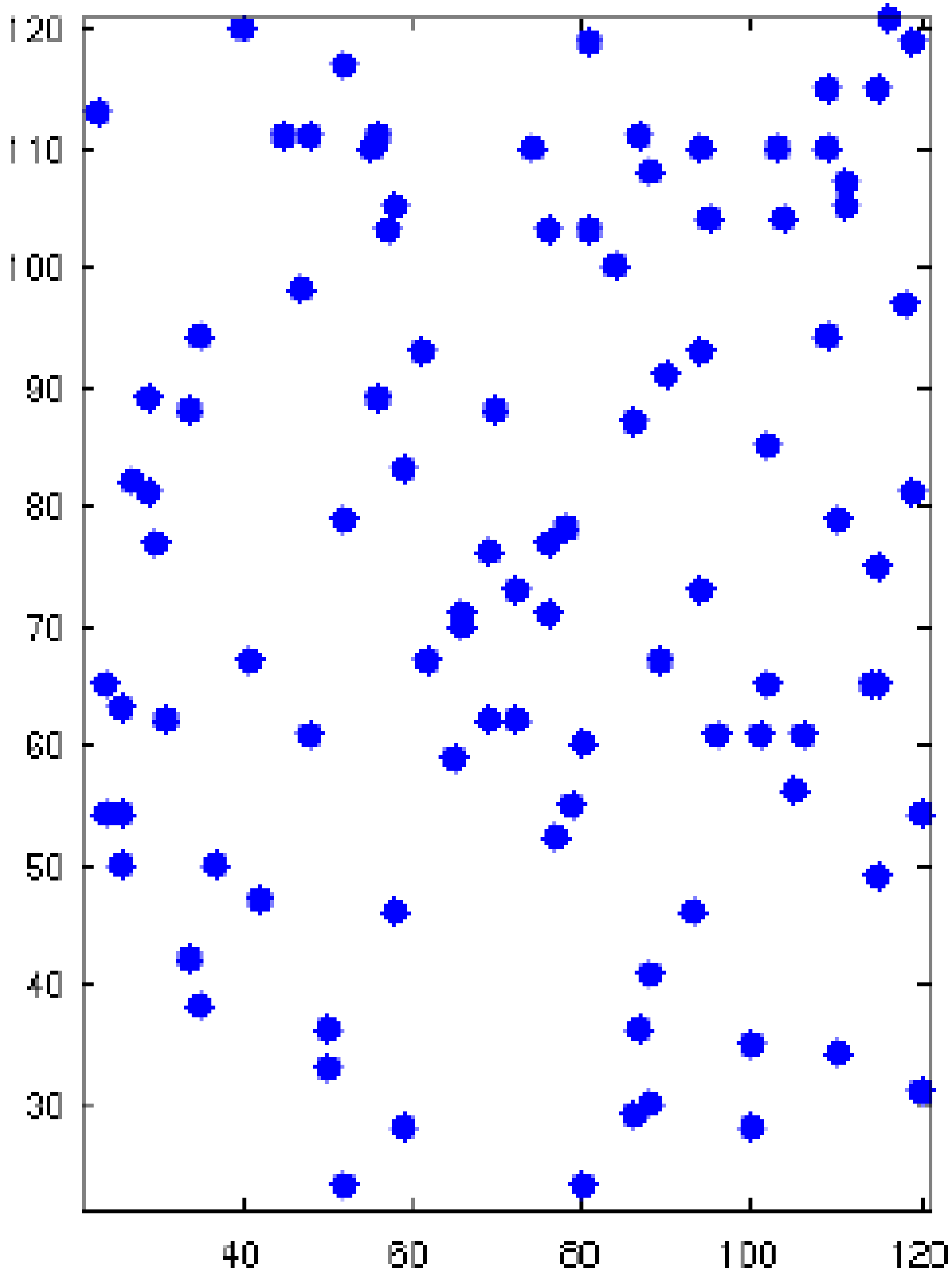


Figure 9: From left to right: original image, result of algorithm 2, result of algorithm 1. Another example. In this case we see that the results for the two algorithms differ. A reason for this is that algorithm 2, because of the counting of occupied square cases, might be favoring alignements of points which are more close together.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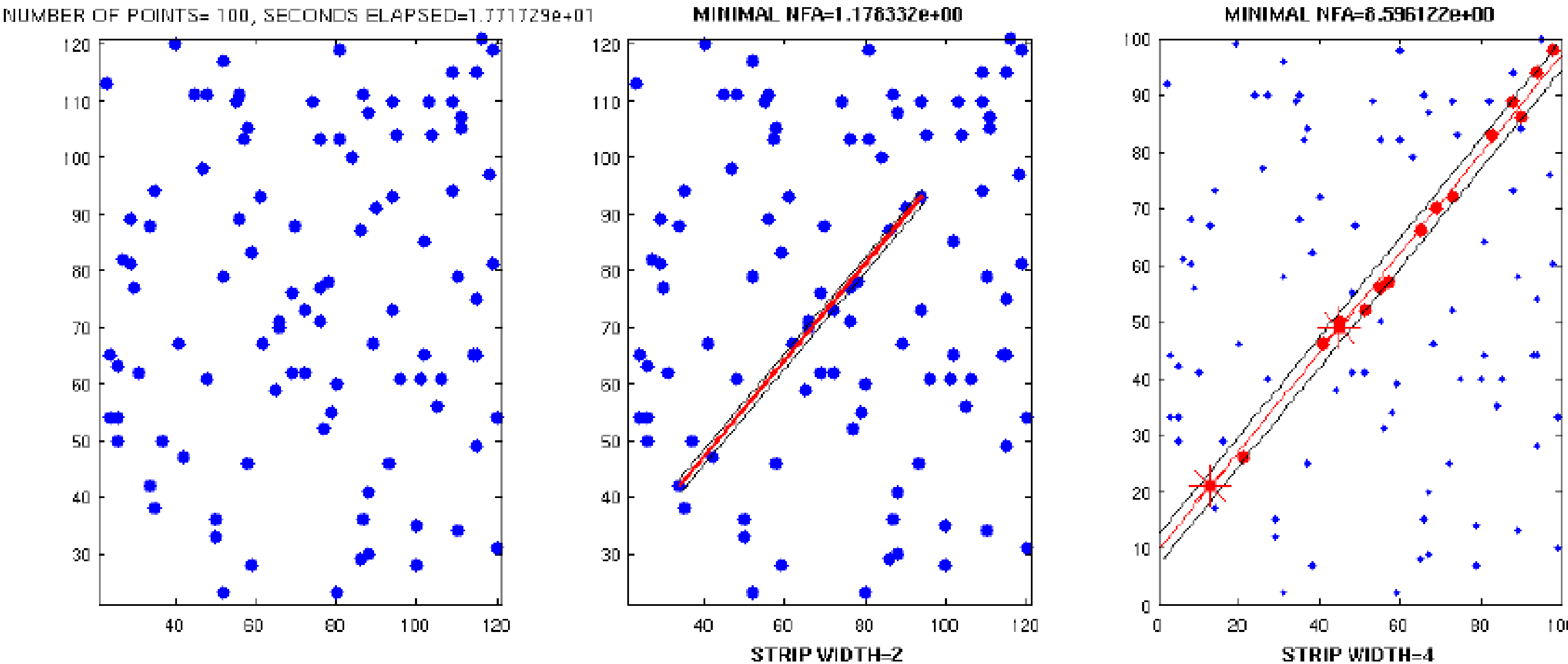

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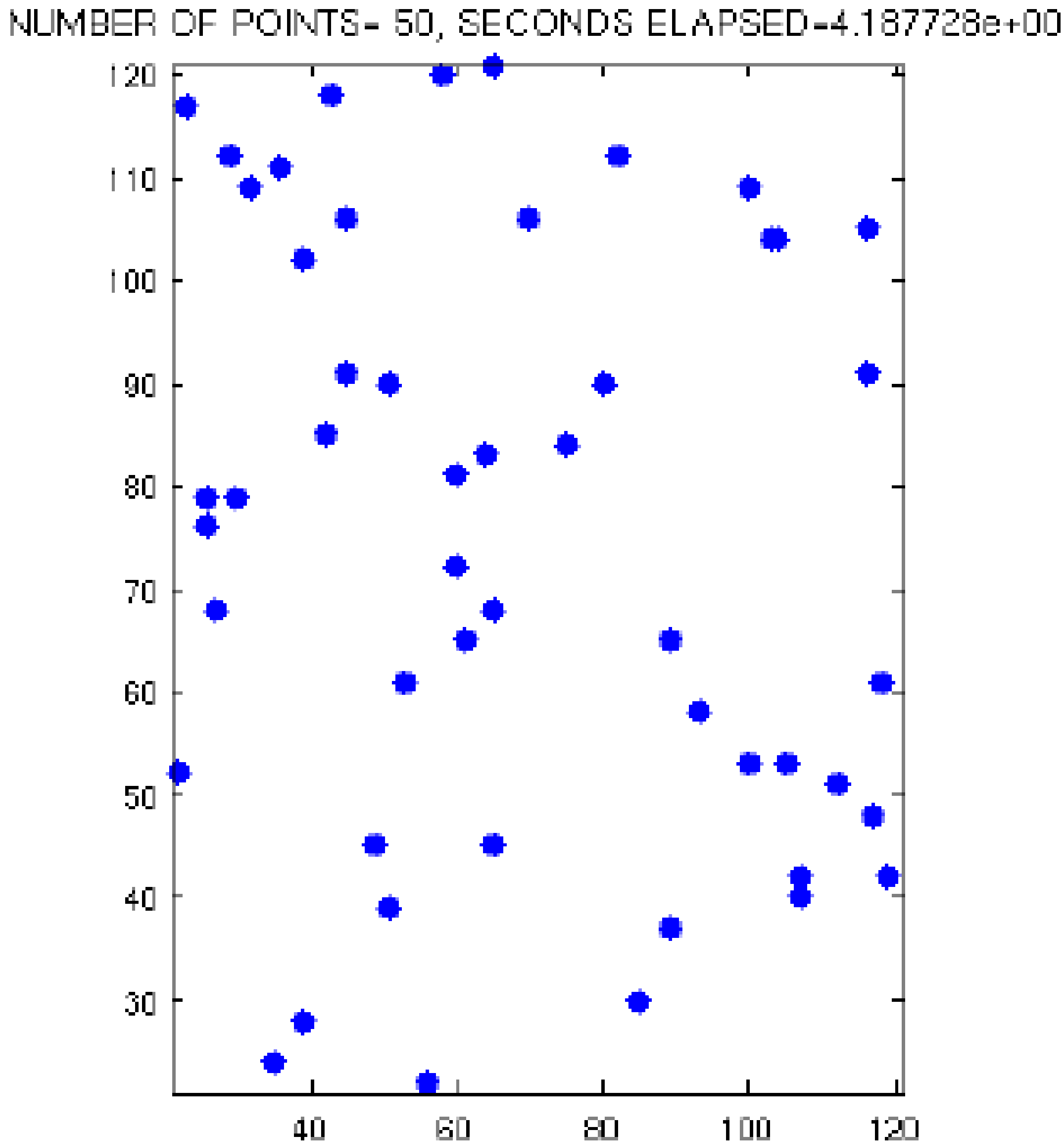
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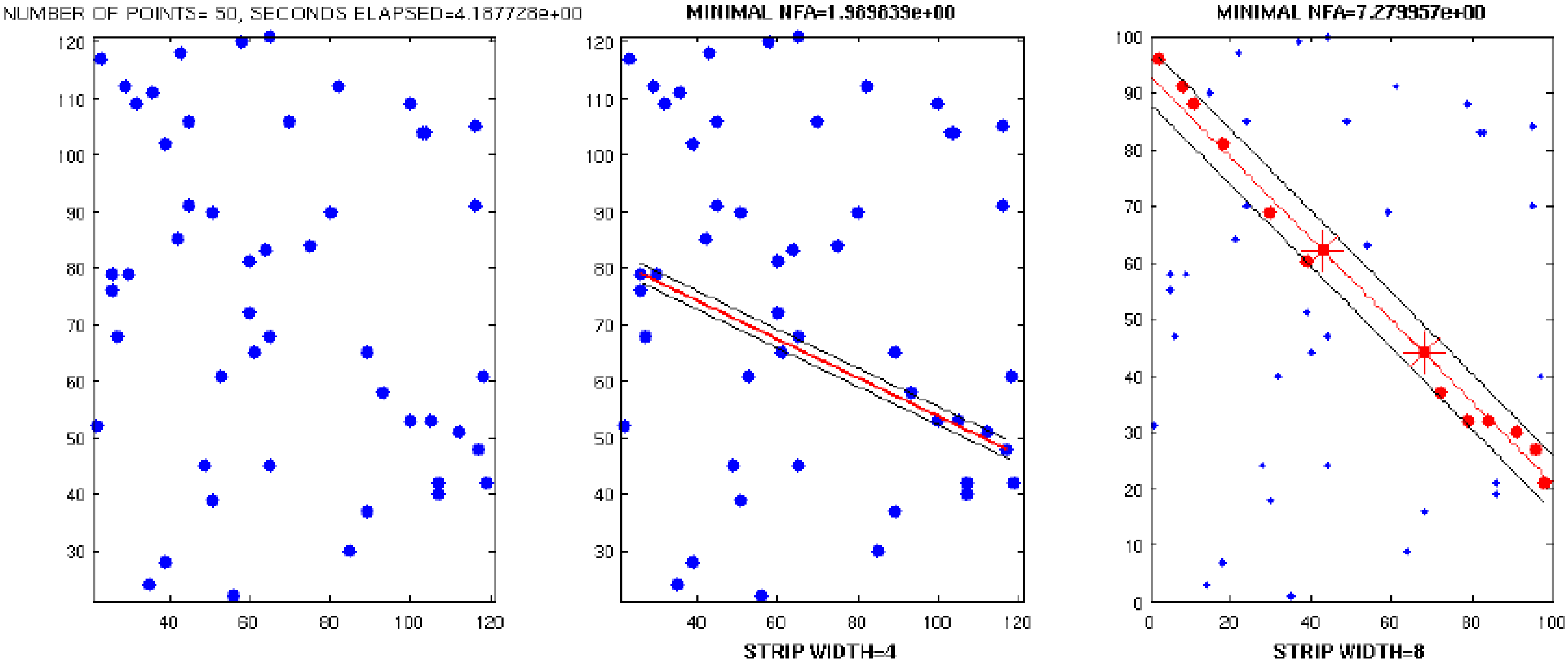

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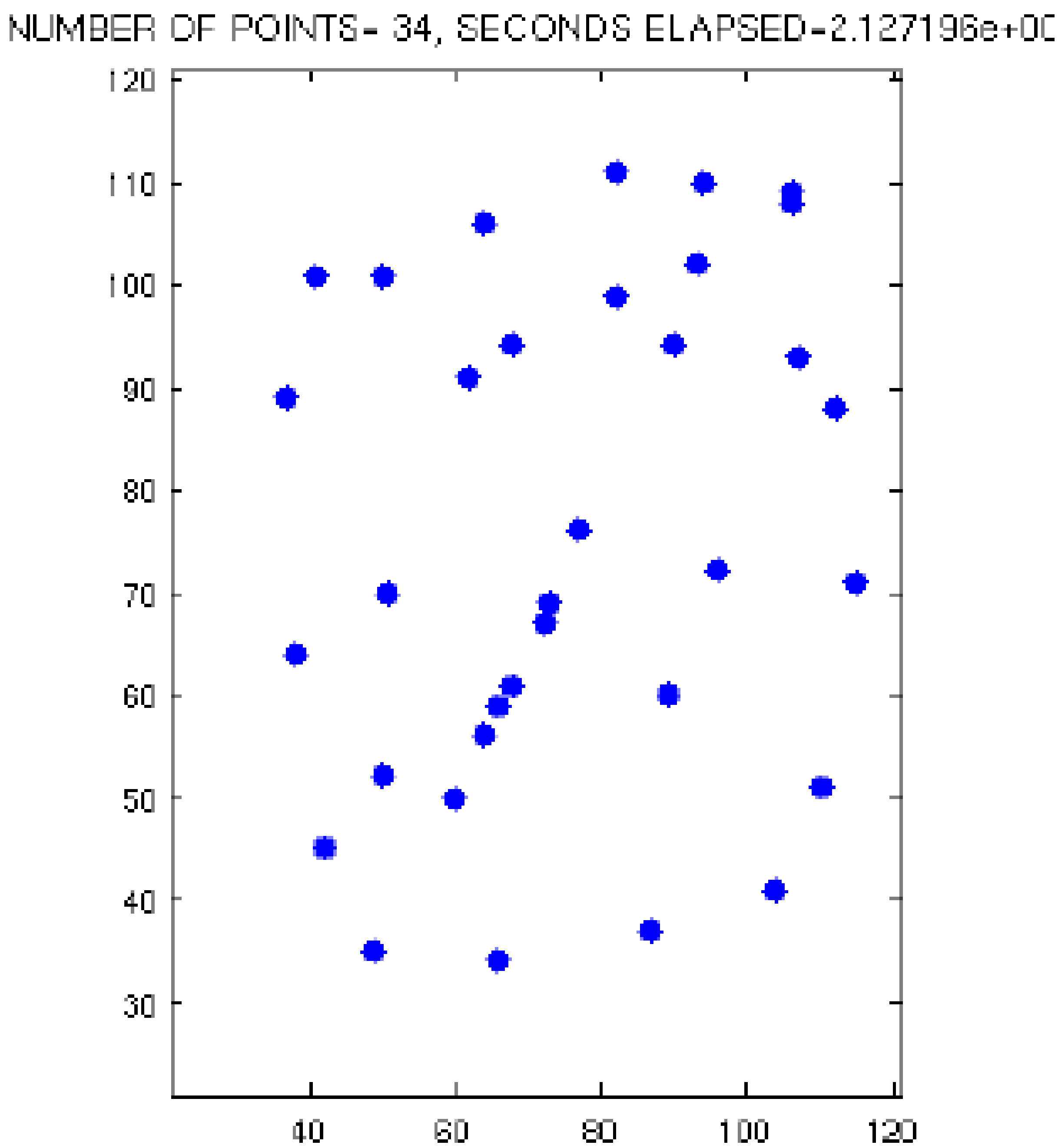
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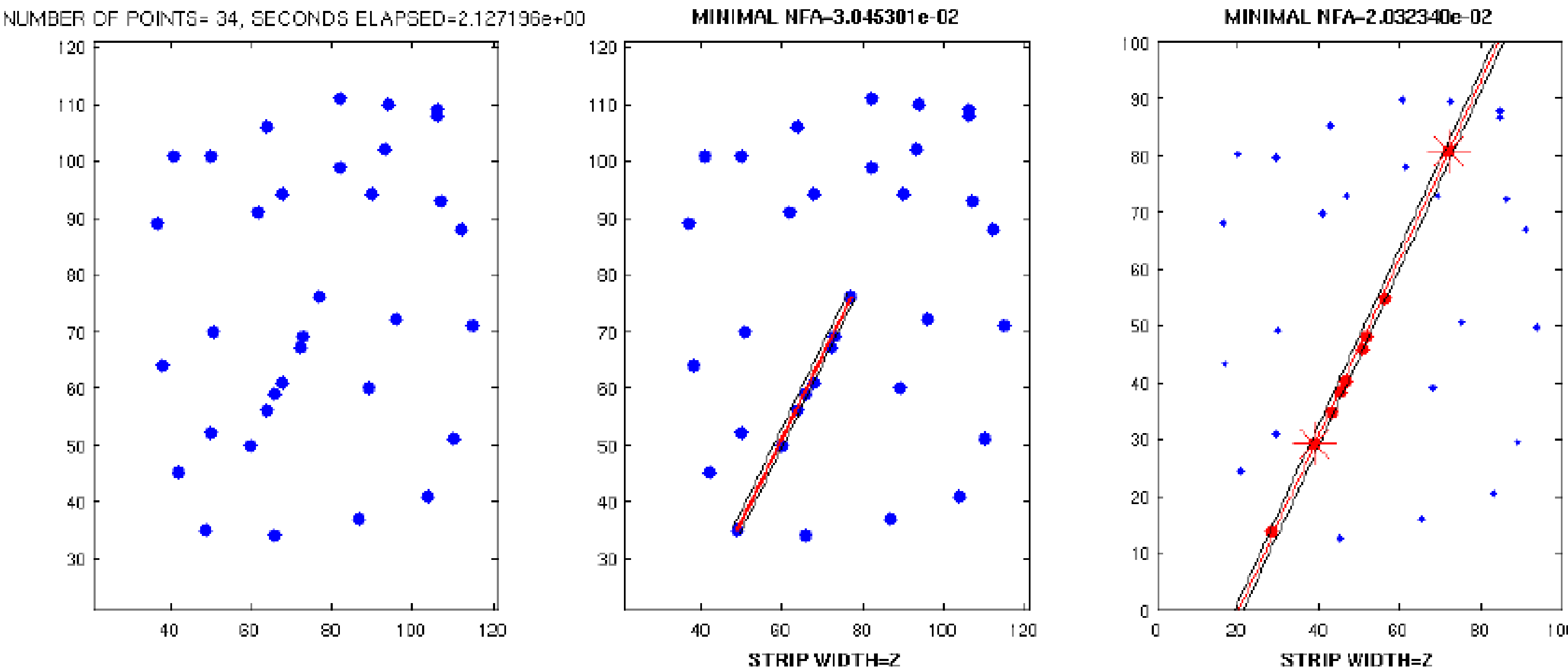

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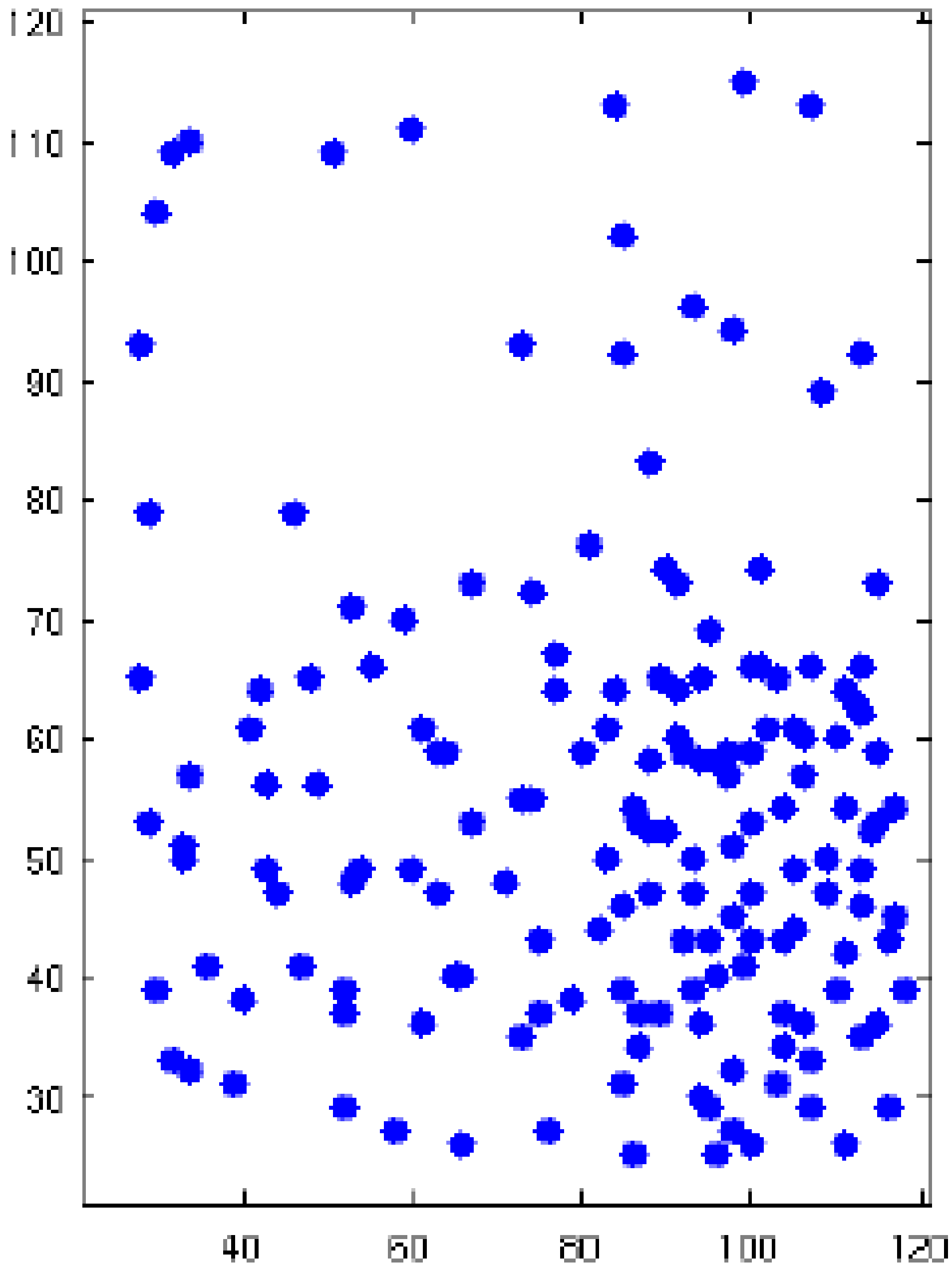

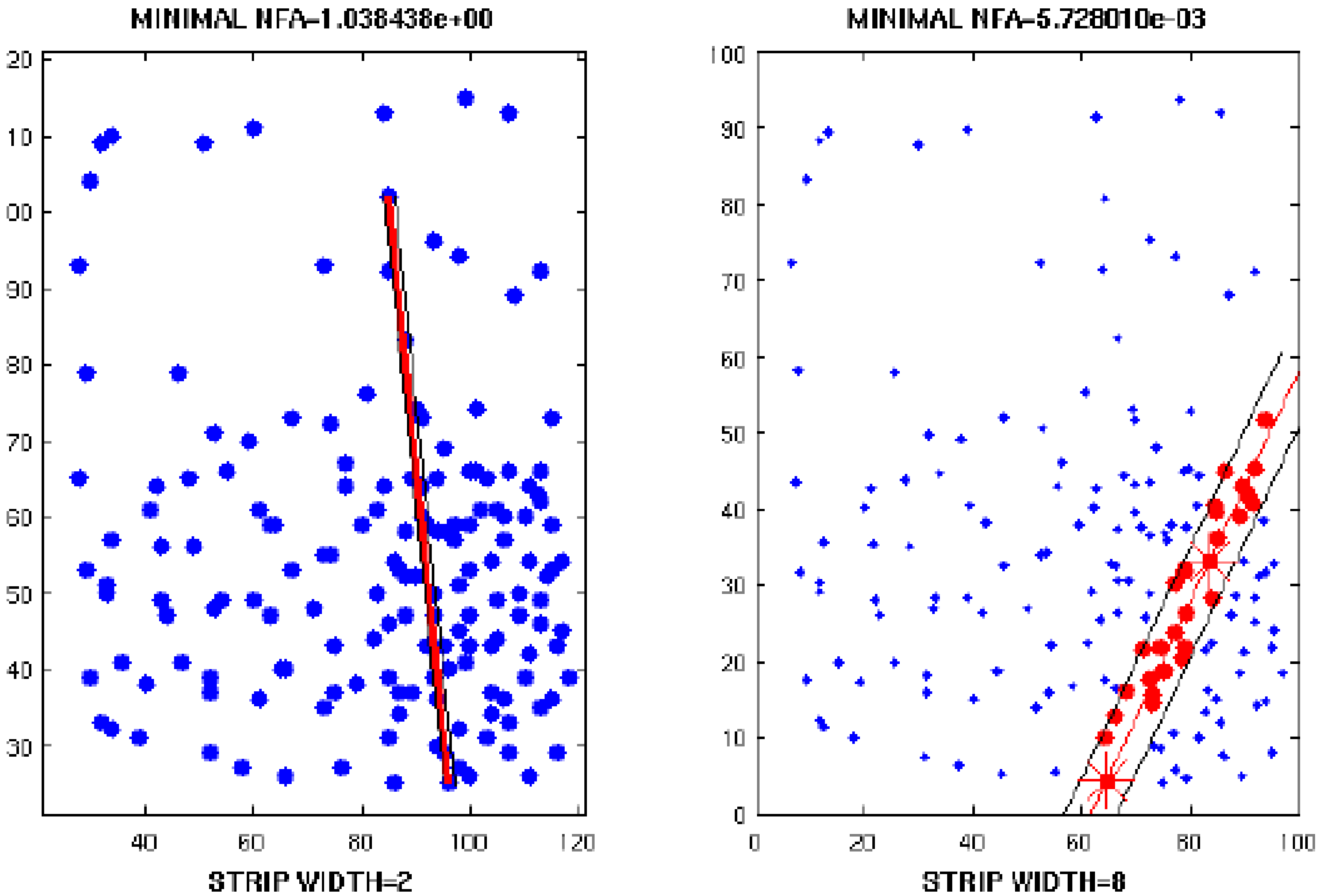

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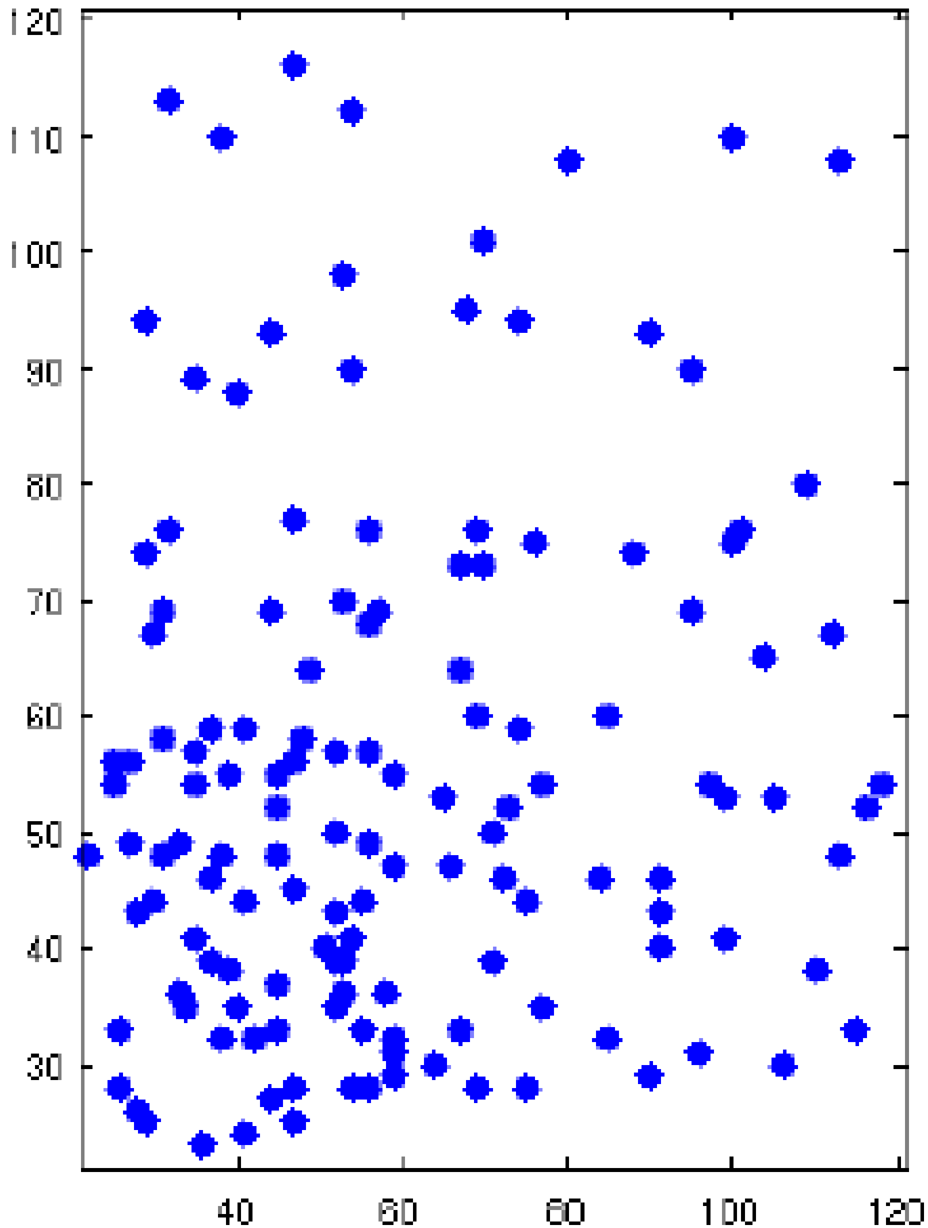

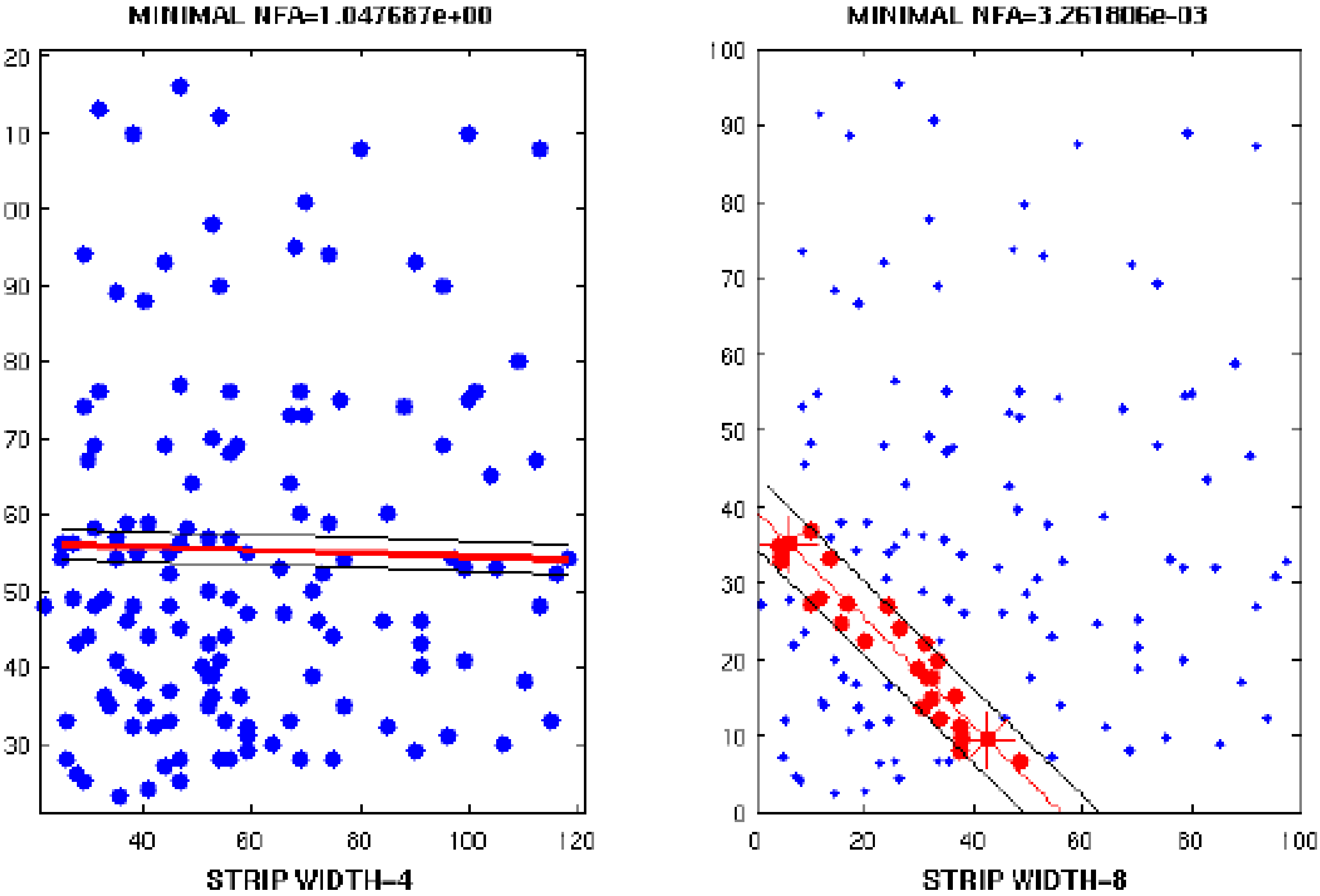

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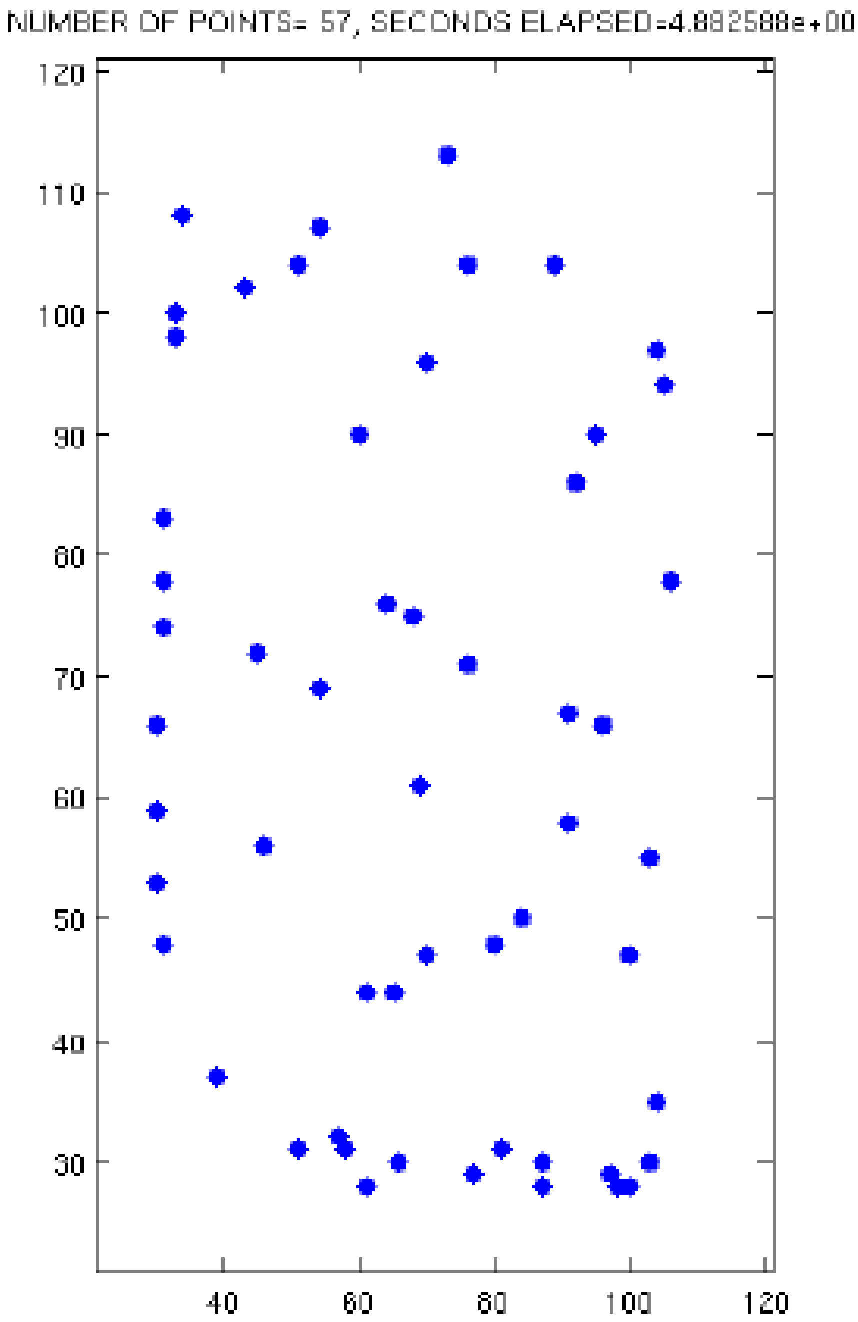
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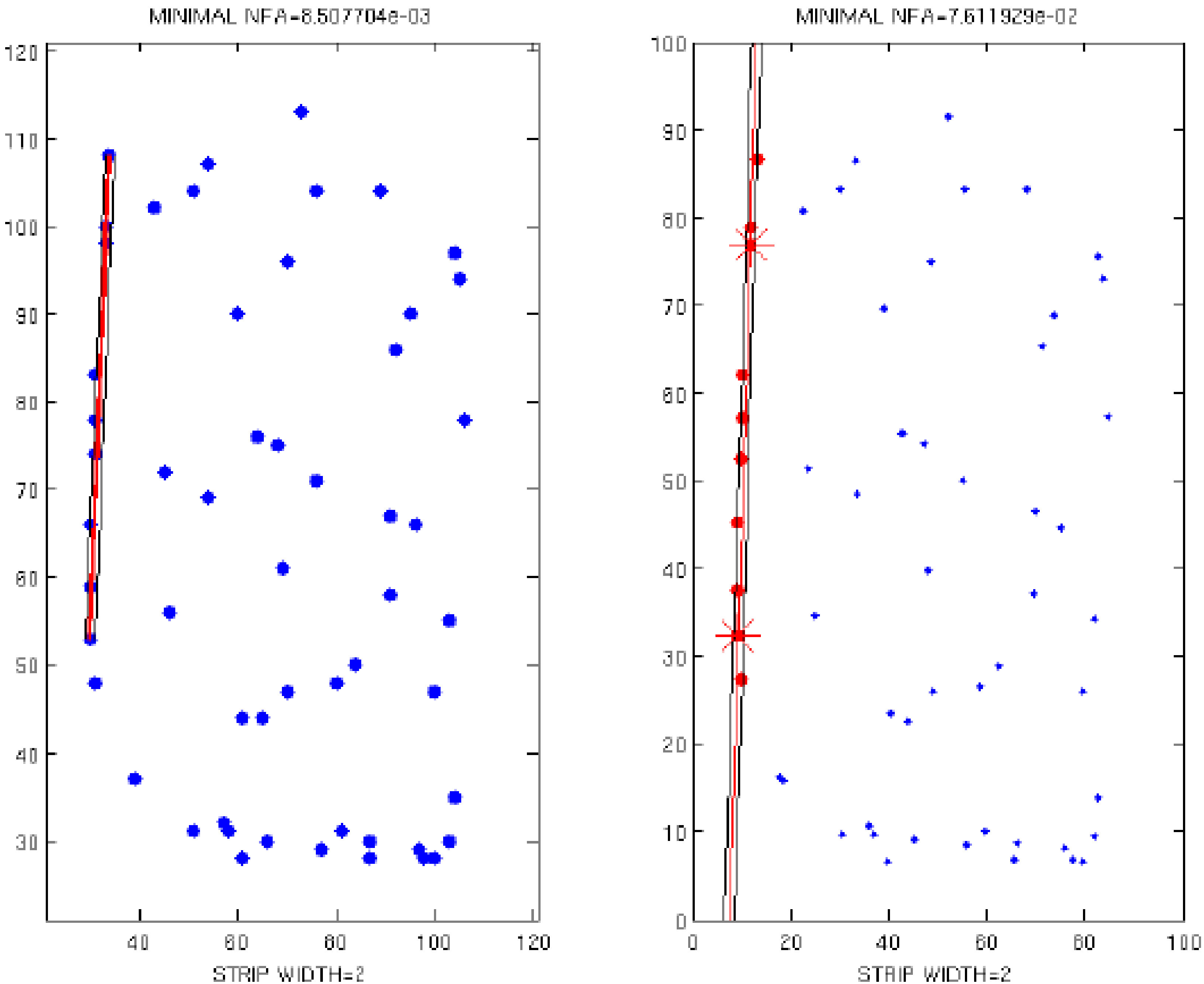
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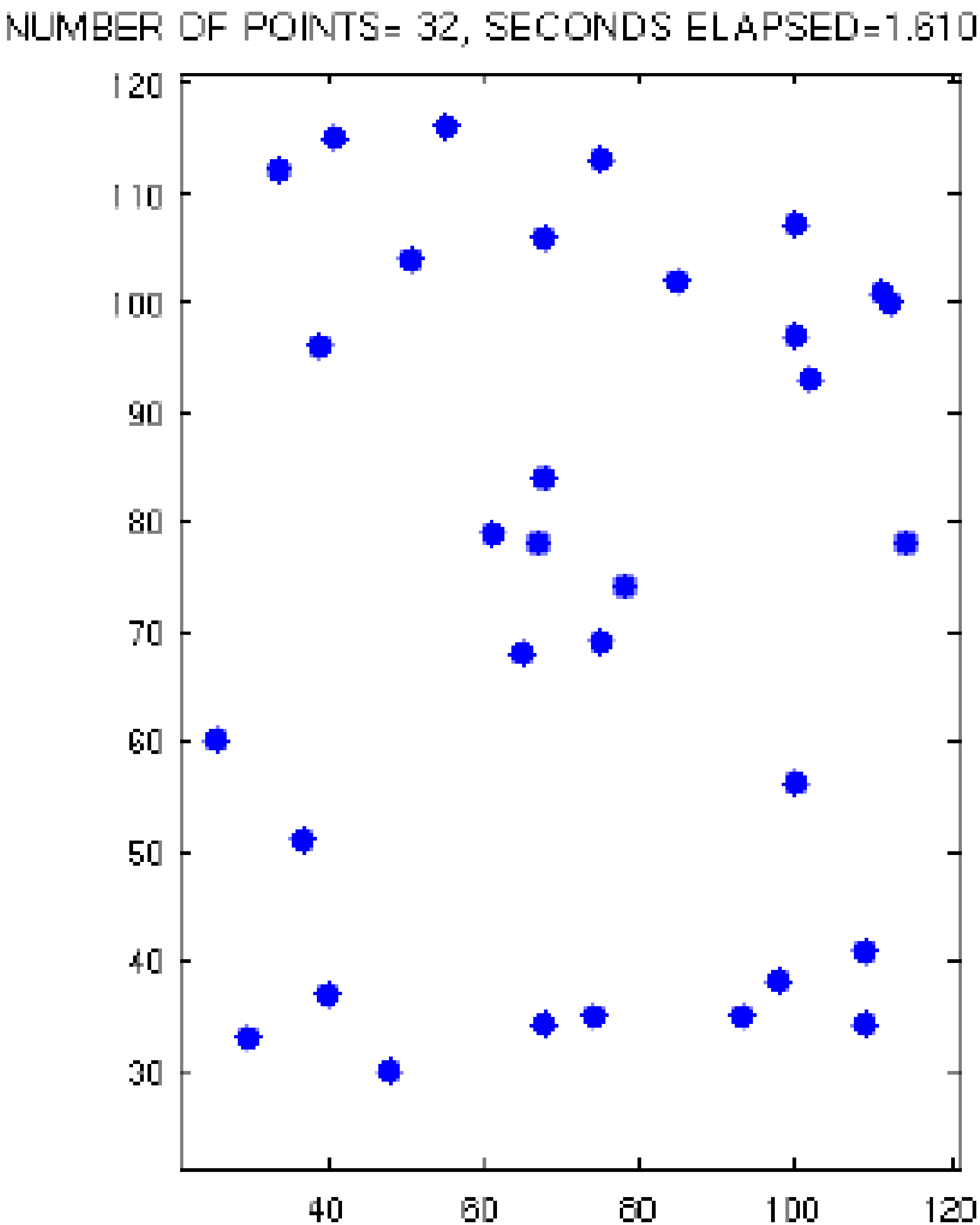
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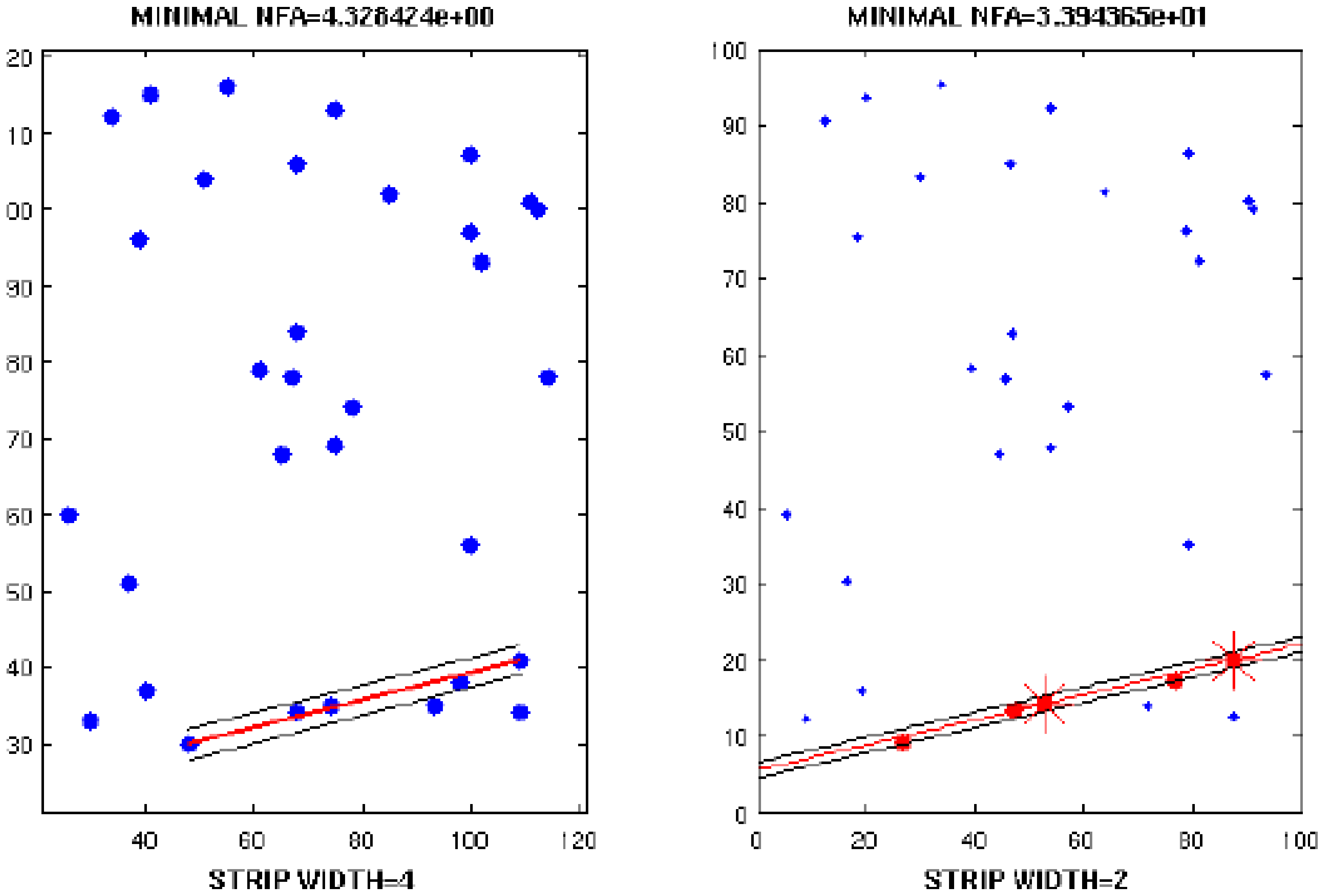

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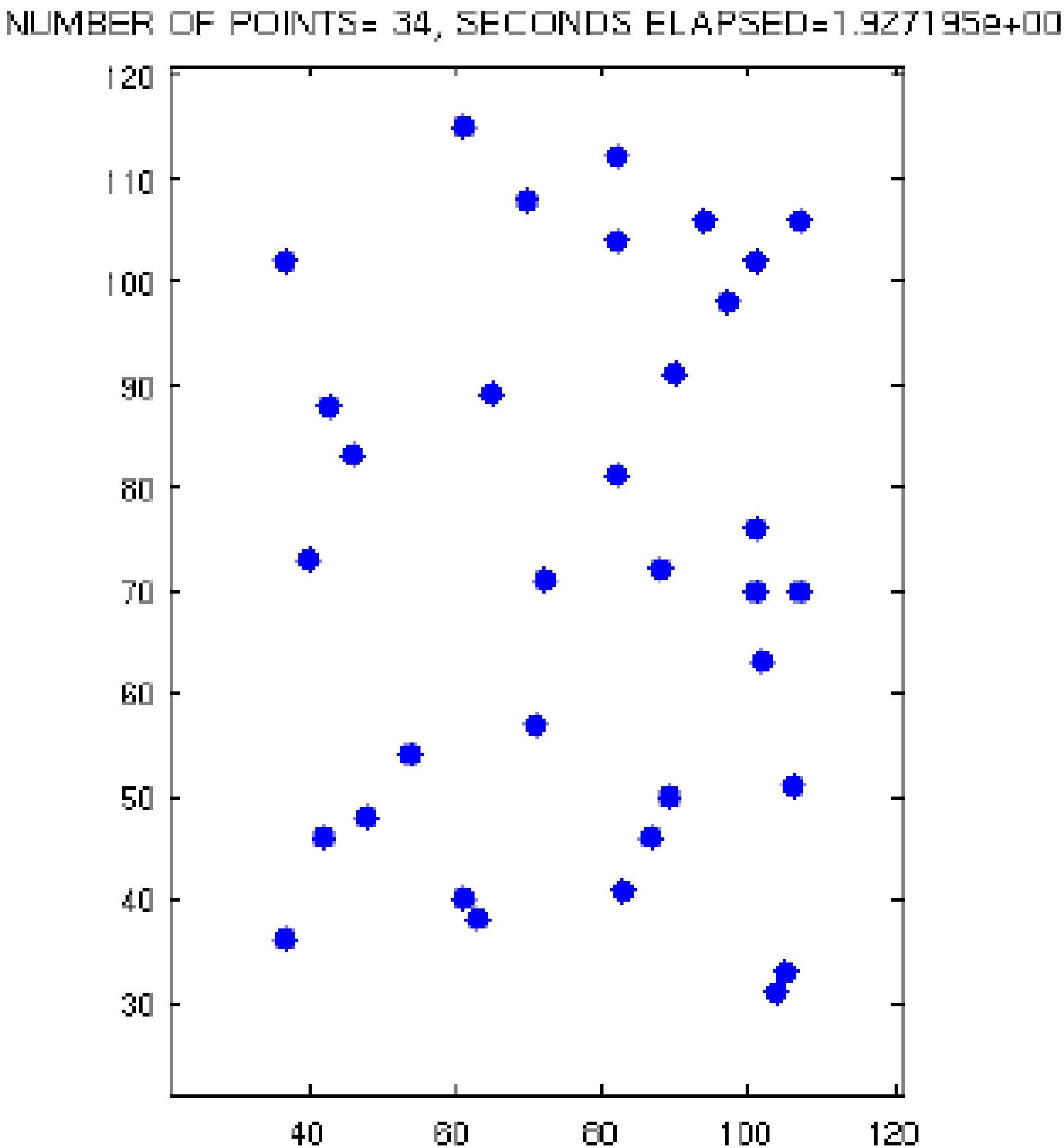
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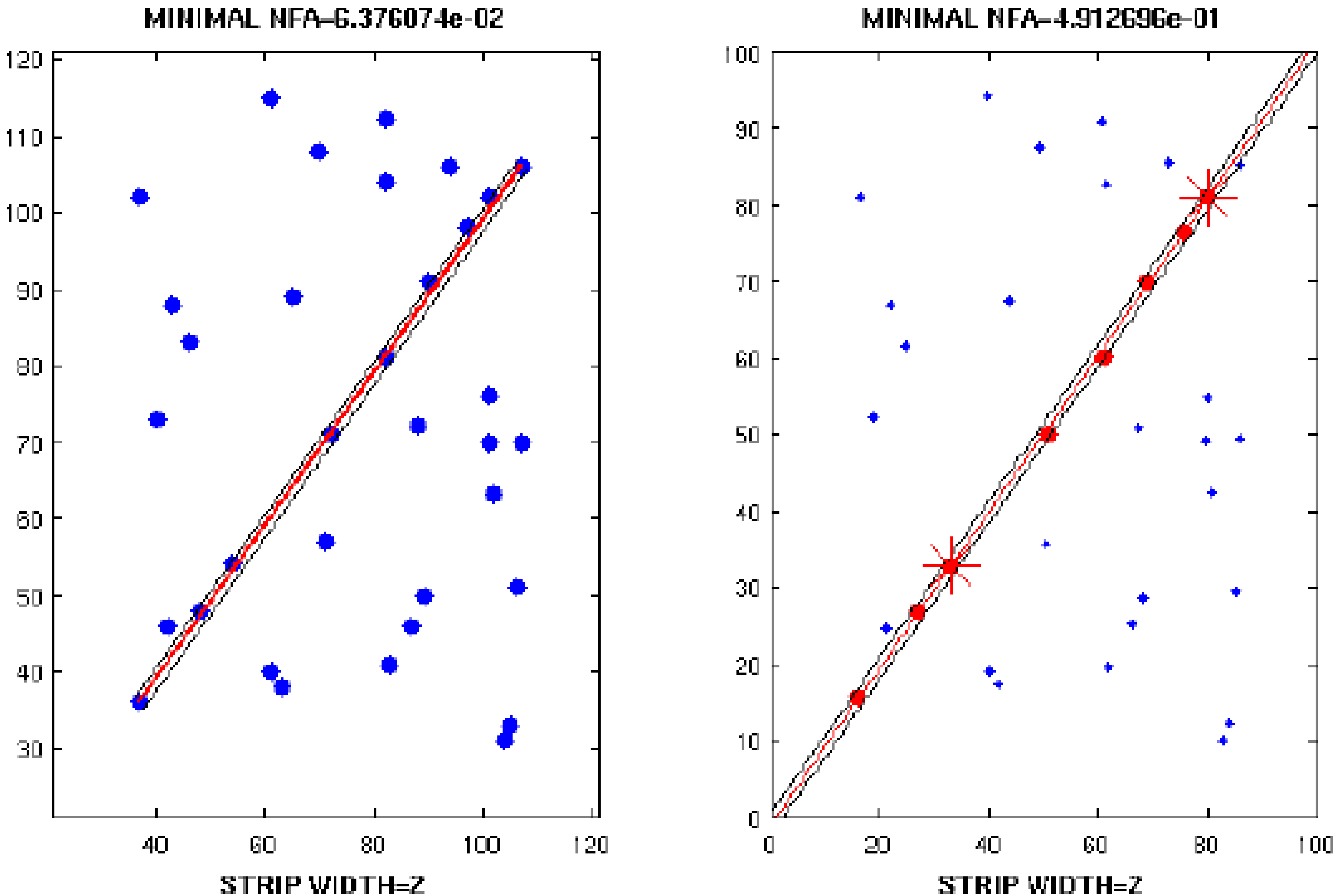
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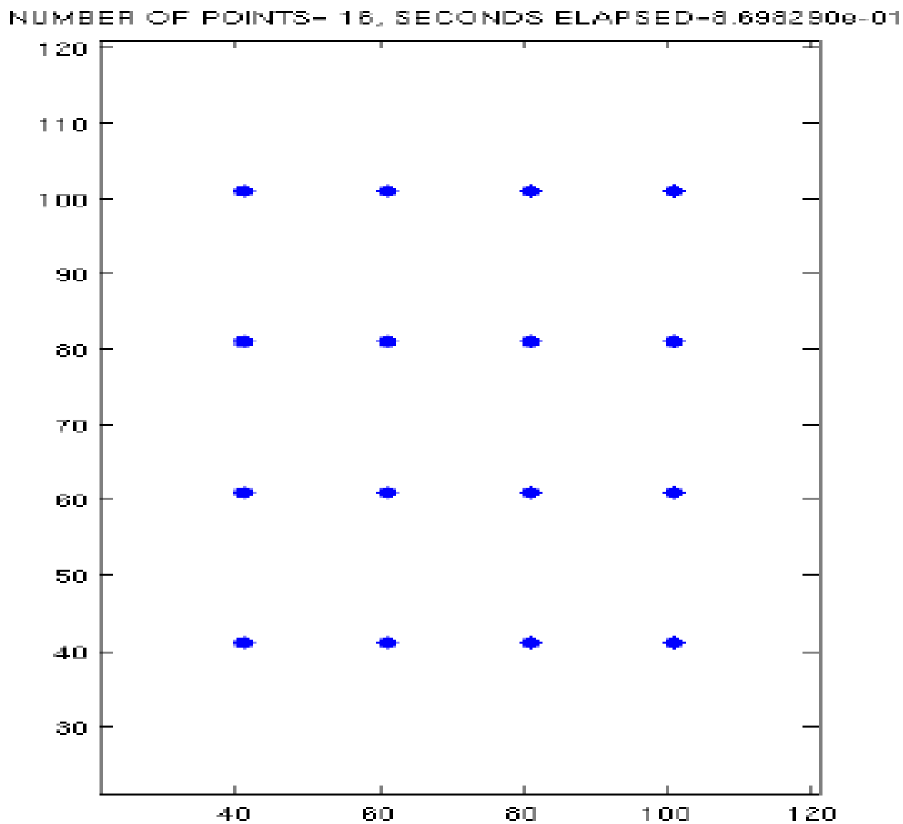

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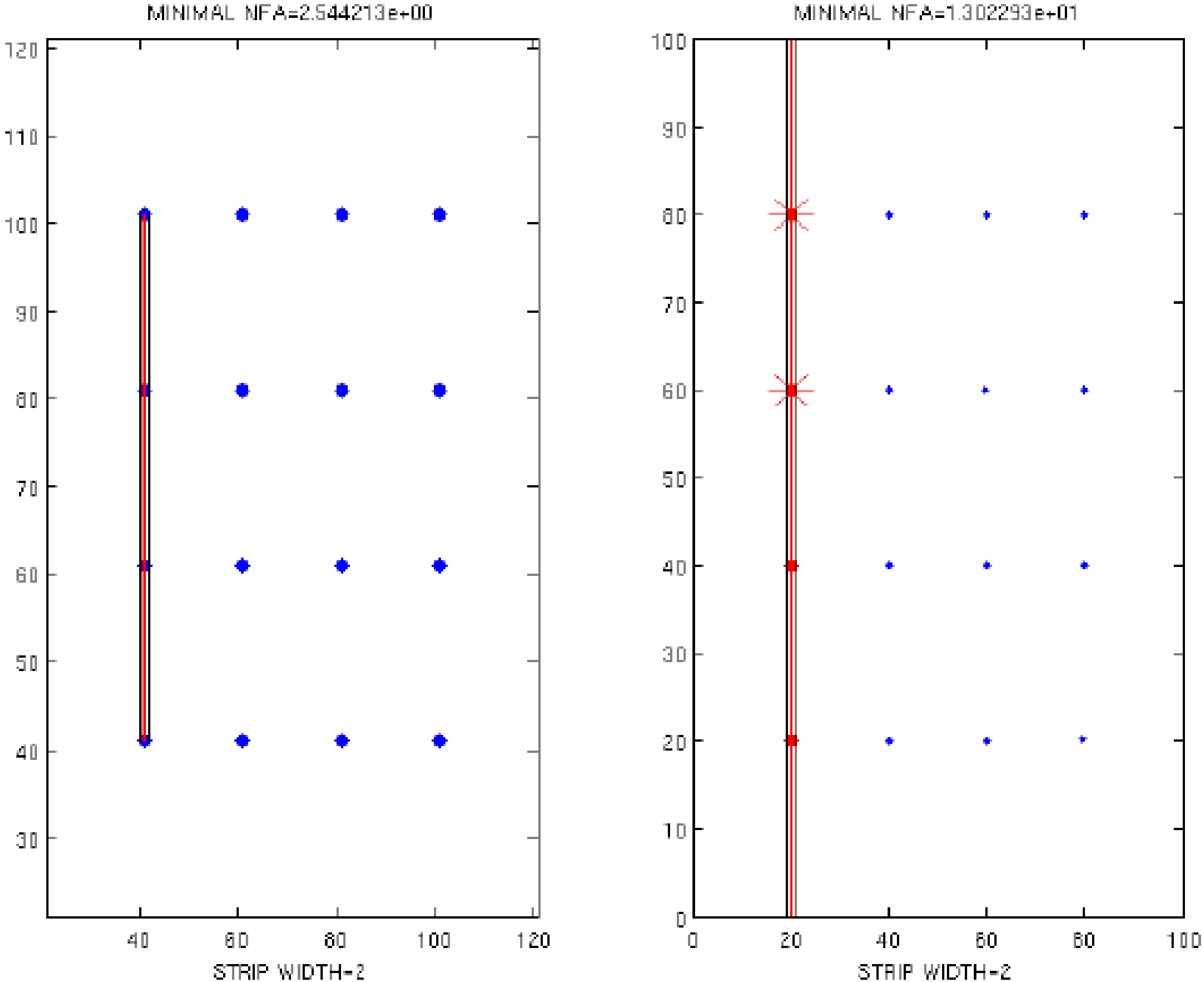
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