

# A contrario block matching

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# What is Stereovision?

## Stereo Principle

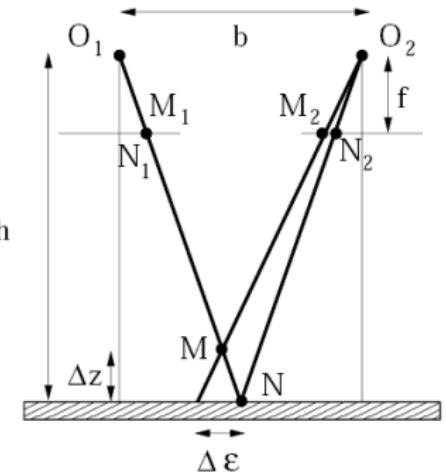
*f* focal length.

*b* distance between centers.

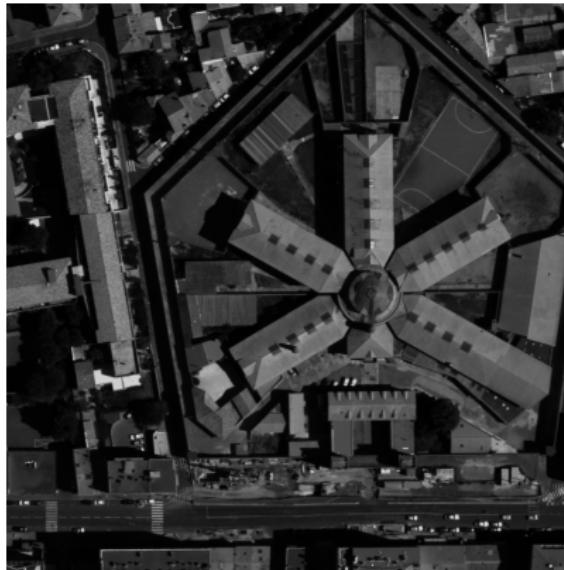
*h* camera-scene distance.

$$\Delta\varepsilon = \frac{M_2 N_2}{f/h}$$

$$\Delta z \simeq \frac{\Delta\varepsilon}{b/h}$$



# Real stereo pair

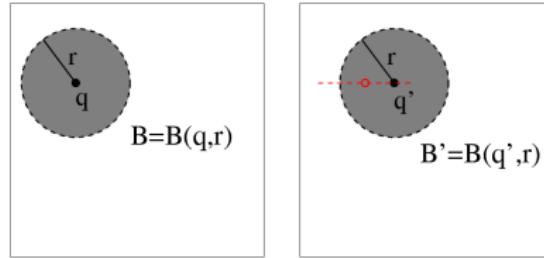


# Real stereo pair



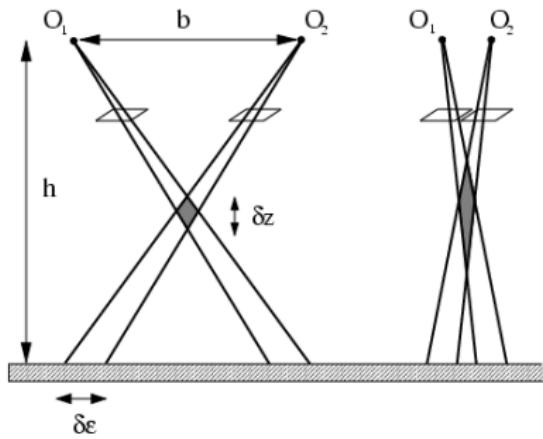
# Block-matching

Given two images  $u$  and  $u'$ , if  $q$  is a pixel of the reference image  $u$  and  $B$  a neighborhood (or block or patch) around that pixel, is there a block  $B'$  in the secondary image  $u'$  that is perceptually equal to  $B$  ?



- Which method for block-matching?  
A similarity measure has to be defined.

# The small baseline case



The bigger  $b/h$ , the more precise the depth measurement.

## Advantages :

- less **occlusions**
- views **quasi-simultaneous**
- less **radiometric changes**

## Challenges :

- More accuracy of  $\Delta\varepsilon$  for the same accuracy of  $\Delta z$

$$\Delta z \simeq \frac{\Delta\varepsilon[\text{pix}]}{b/h} R[m/\text{pix}]$$

# CNES' MARC software

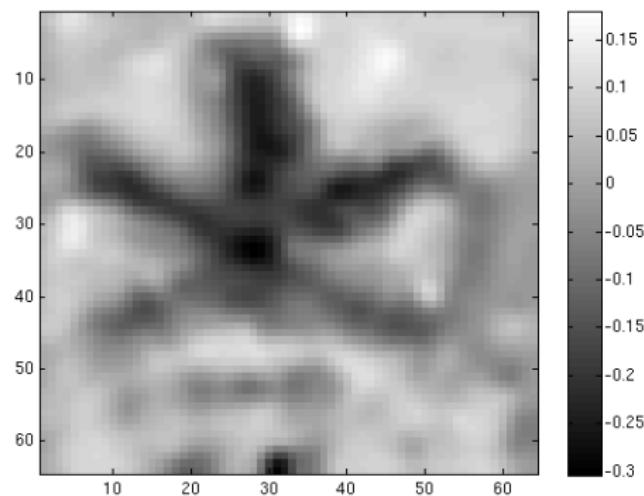
## Strengths and weaknesses :

- Excellent **subpixel localization** of individual matches (small baseline case)
- Good **disparity error estimate**  
(...provided the correct local maximum is detected!)
- Only partial correction of **adhesion effects**
- Only partial elimination of **false correspondences**  
(at the price of strong regularization (smoothing) of the elevation map)

# MARC's approach to avoid false correspondences

## 1. Multiscale structure to avoid false local minima

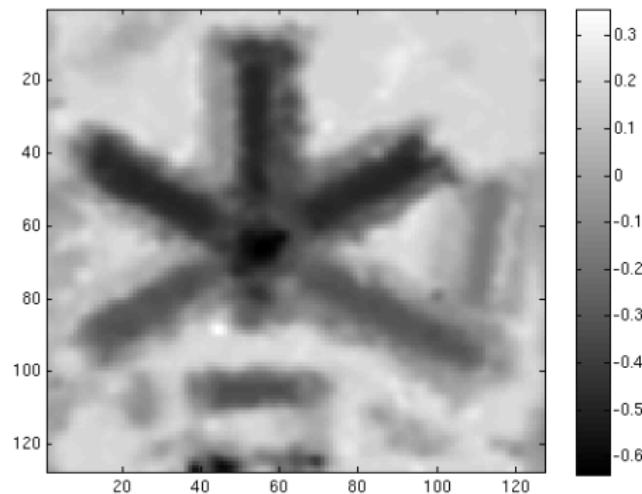
scale 3 (1 graylevel = 4 pixel disparity)



# MARC's approach to avoid false correspondences

## 1. Multiscale structure to avoid false local minima

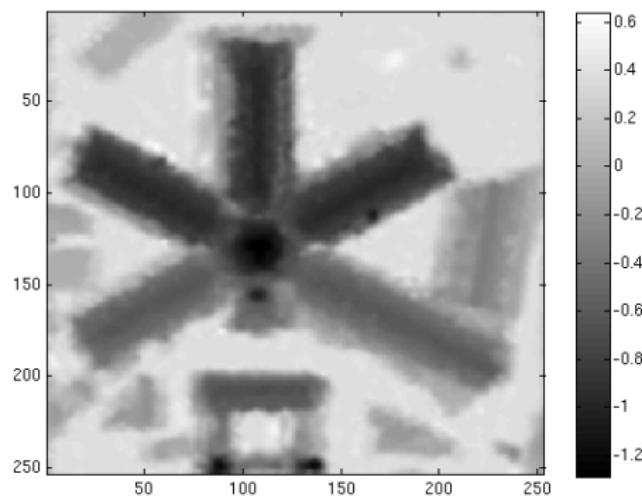
scale 2 (1 graylevel = 2 pixel disparity)



# MARC's approach to avoid false correspondences

## 1. Multiscale structure to avoid false local minima

scale 1 (1 graylevel = 1 pixel disparity)

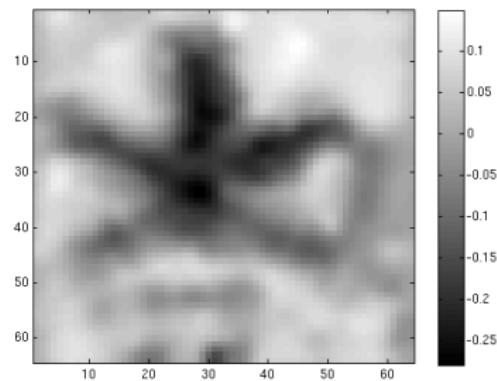


# MARC's approach to avoid false correspondences

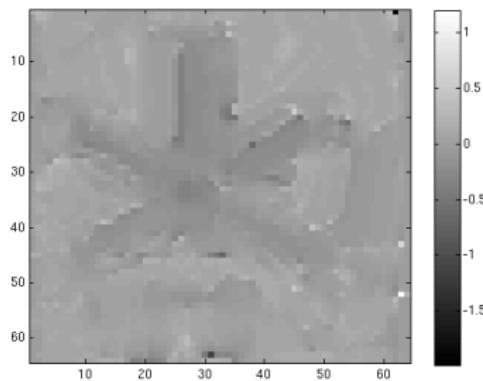
2. At each scale smoothing interpolation is used

scale 3 (1 graylevel = 4 pixel disparity)

after smoothing interpolation



before smoothing interpolation



# Approach & Methodology

## Our approach

- **Eliminate false correspondences** before regularization/interpolation
- **Detect geometrical 3D structure** from this point cloud + image information (planes, edges, curved surfaces)
- regularize/interpolate based on detected structure

## Methodology

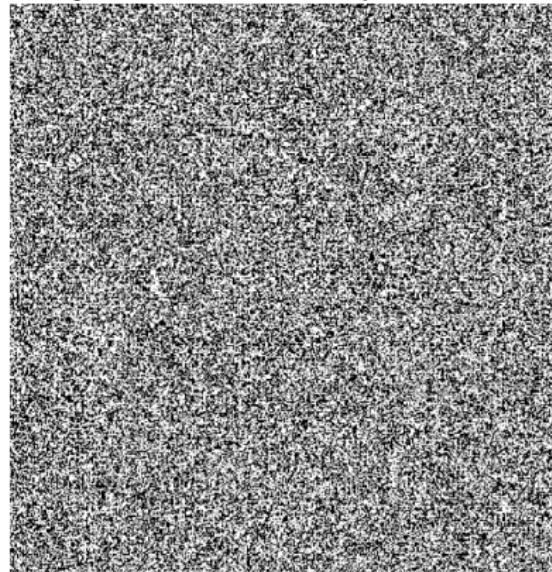
**A contrario detection** to elliminate false detections

# A contrario detection methods

- Inspired on **Gestalt theory** of visual perception  
⇒ “qualitative” laws of perceptual grouping
- Augmented with **sampling** theory and **probabilistic** approach  
⇒ mathematical and computational framework
- Compute **automatic detection thresholds** using common sense principle “no detection in white noise ”
- **Helmholtz principle**  
*A grouping  $G$  of objects having a common characteristic is perceptually significant, if the expected number of occurrences of such an event by chance is very small*
- What do we mean by “chance” ? This is the “a contrario” model. The least structured statistical model that can be reasonably conceived. Usually white noise.

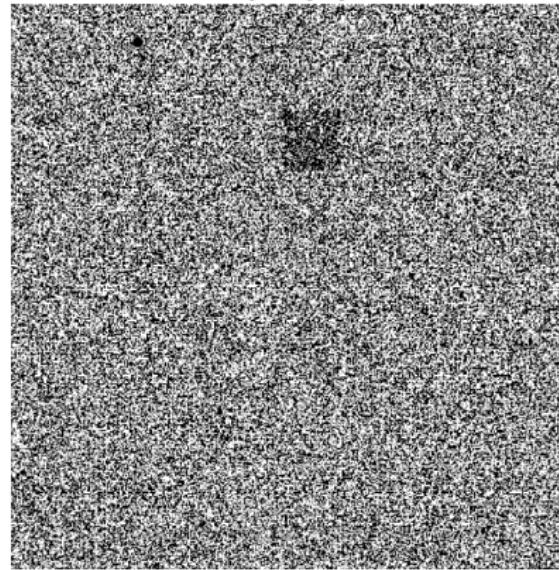
# A toy example

Do you see a dark square here ?



# A toy example

And here ?



# Which a contrario model for block-matching ?

- **Point matching is hopeless** (few detections)

$$NFA(q, q') = N_{test} * P(|u(q) - u'(q')| < \theta) = \frac{N_{tests}}{SNR} < 1 \text{ (for } \theta = \sigma\text{)}$$

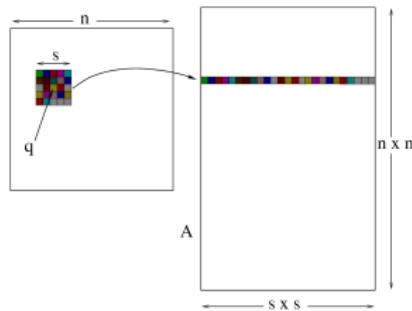
- Some **neighborhood information** is needed

$$NFA(q, q') = N_{test} * P\left(\frac{1}{B(0)} \sum_{d \in B(0)} |u(q + d) - u'(q' + d)|^2 < \theta^2\right)$$

Which (a contrario) statistical model for image patches  $u'|_{B(q')}$  ?

- **learning empirical distribution?** hopeless! (very high dimensional space)
- learn some **marginals** and assume **independence**
  - **independent pixels**: too naïve! leads to false matches
  - **independent PCA coefficients** : OK!

# Extracting neighborhood features : PCA

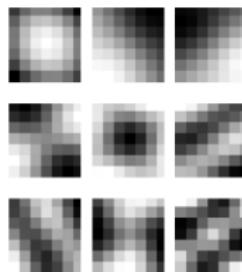


- An  $n \times n$  image contains  $\approx n^2$  (overlapping)  $s \times s$  blocks  $B$
- The grayvalues  $u(B)$  are considered as instances of a random  $s^2$ -vector  $X_1, \dots, X_{s^2}$
- Matrix  $A$  stores the  $n^2$  instances of  $X_1, \dots, X_{s^2}$
- Build the covariance matrix  $C = \text{Cov}(A)$ . The eigenvectors of  $C$  provide the Principal Component (PC) basis.
- The projections of a block  $u(B)$  in the direction of each eigenvector provides the PC coefficients.

# PCA analysis of $7 \times 7$ blocks



(a)



(b)

Figure 2: (a) Reference Image  $I$ . (b) The nine first principal components with larger eigenvalues of  $I$ , for  $7 \times 7$  neighborhoods.

# PCA analysis of 7x7 blocks

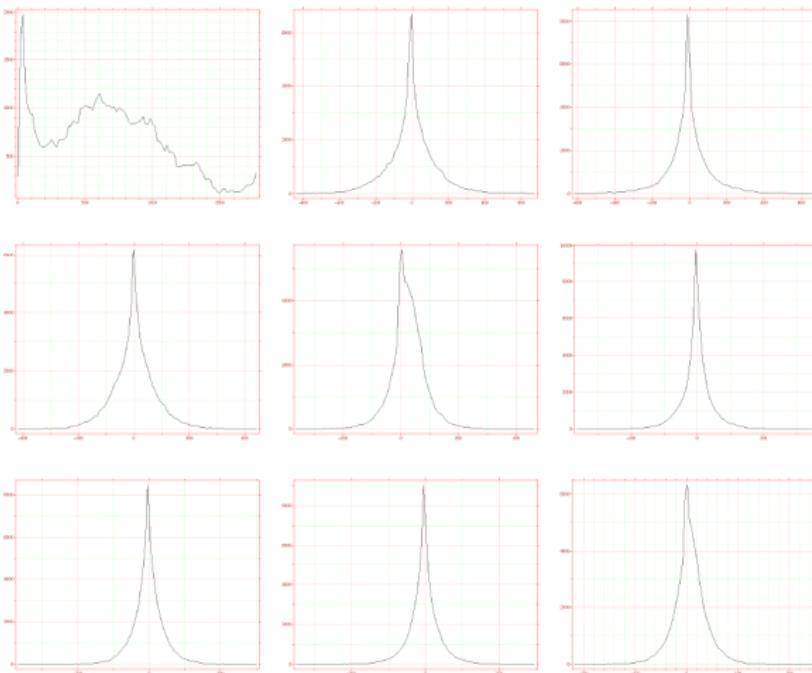
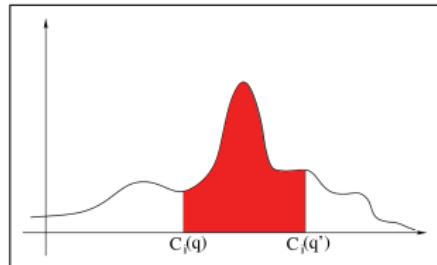


Figure 3: Histograms of the nine first principal components.

# Similarity measure

- Choose the  $N$  PC's having the largest eigenvalues among the  $s^2$ .
- Let  $c_i(q)$  be the value of the  $i$ -th PC coefficient at point  $q$ ,  $i = 1, \dots, N$ .
- Let  $H_i(q) := H_i(c_i(q))$  be the cumulative histogram of  $c_i(q)$  for the secondary image.



## Definition (Empirical Probability of coefficient similarity)

Given a stereo pair of images  $u$  and  $u'$  and  $q \in u$ ,  $q' \in u'$ . For every  $i = 1, \dots, N$  the empirical probability that an observed block  $B_{q'}$  in  $u'$  be similar to  $B_q$  for the feature  $i$  ,

$$\widehat{p}^i(B_q, B_{q'}) = 2 \cdot |H_i(q) - H_i(q')|.$$

# Similarity measure

- Consider a family of  $Q$  probability thresholds:

$$p_j = \left(\frac{1}{2}\right)^{j-1}; \quad j = 1, \dots, Q.$$

Definition (Increasing probability distribution upper bound)

For a pair of points  $q$  and  $q'$ , and for all  $i = 1, \dots, N$ ,

$$p_{q,q'}^i = \inf_j \{ p_j \mid p_j \geq \widehat{p_{q,q'}^i}, p_j \geq p_{q,q'}^{i-1} \text{ si } i \geq 2 \}.$$

Definition ( $\varepsilon$ -meaningful block-match)

A pair of points  $q$  and  $q'$  is an  $\varepsilon$ -meaningful block-match if  $NFA_{q,q'} \leq \varepsilon$ , where

$$NFA_{q,q'} = N_{test} \prod_{i=1}^N p_{q,q'}^i.$$

# Similarity measure

- Consider a family of  $Q$  probability thresholds:

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# Computation of the number of tests

- The image size:  $n^2$ .
- The size of the search domain for the second point  $q'$ :  $n_{epipol}$ .
- The number of possible increasing quantized probability distributions  $p_{q,q'}^i$  with  $N$  PC's and  $Q$  probability thresholds:  $FC_{N,Q}$ .

$$N_{test} = n^2 n_{epipol} FC_{N,Q}$$

$$FC_{N,Q} = \#\{f : \llbracket 1, N \rrbracket \rightarrow \llbracket 1, Q \rrbracket \mid f(x) \leq f(y), \forall x \leq y\}$$

$$FC_{N,Q} = \sum_{t=0}^Q (t+1) \binom{N+Q-t-3}{Q-t-1}$$

# Shadows

A shadow seems homogeneous from the global contrast perspective  
but expanding contrast reveals some local structure.



Conclusion : Local PCA to avoid false detections in shadows.

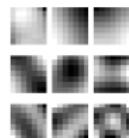
# Local PCA analysis of 7x7 blocks



Image partition into 4 classes based on mean and variance of the patches:



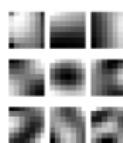
(b1)



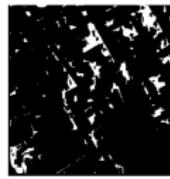
(b2)



(d1)



(d2)



(c1)



(c2)



(e1)



(e2)

## Self-similarity threshold

Given a meaningful block match  $(q, q')$  between images  $u$  and  $u'$ .  
The match is **rejected** if there is a point  $r \in u$  near  $q$  such that

$$d_{L^2}(B_q, B_r) < d_{L^2}(B_q, B_{q'})$$

This sanity check avoids "**stroboscopic effects**" (very large disparity errors due to self-similar repeated structures) at the cost of a slightly sparser disparity map.

# Précision subpixellique : Raffinement par corrélation

On détermine la disparité  $\mu^d$  sur chaque point  $x_0$ :

$$\mu^d(x_0) := \arg \min_{\mu} e^d(\mu, x_0), \quad (1)$$

$$e^d(\mu, x_0) := \|\tau_\mu u - \tilde{u}\|_{\varphi_{x_0}}^2. \quad (2)$$

où

- $\tau_\mu u(x) = u(x + \mu)$ ,
- $\varphi_{x_0} = \varphi(x - x_0)$  fenêtre symétrique à support compact.
- $\|u\|_{\varphi_{x_0}}^2 = \langle u, u \rangle_{\varphi_{x_0}}$ ,
- $\langle u, v \rangle_{\varphi_{x_0}} = (\frac{1}{2})^2 \sum_{x \in \frac{1}{2}\mathbb{Z}^2} u(x)v(x)\varphi_{x_0}(x)$ .

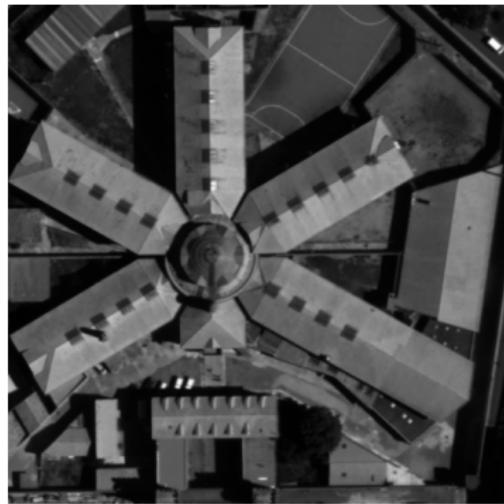
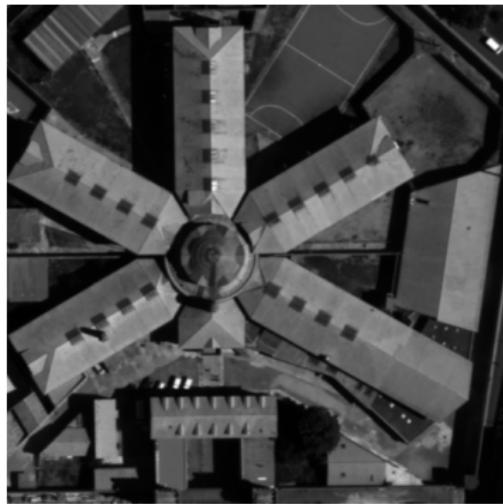
Subpixel discrete correlation requires x2 zoom.

# Causes de l'Erreur.

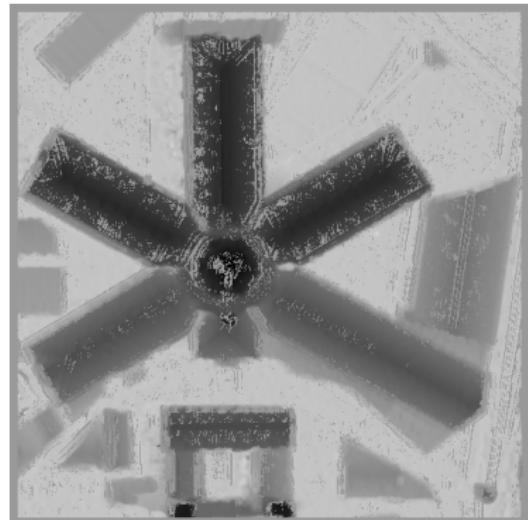
$$\mu^d(x_0) = \varepsilon(x_0) + \underbrace{A(x_0)}_{\text{adhesion error}} + \underbrace{r(x)}_{\text{noise error}} + \underbrace{\mu^d(x_0) - \mu^c(x_0)}_{\text{discretization error}}, \quad (3)$$

- Gross errors ( $\sim 10$  pixels).
- Adhesion errors ( $\sim 1$  pixels).
- Noise errors ( $\sim 0.1$  pixels).
- Discretization errors ( $\sim 0.01$  pixels).

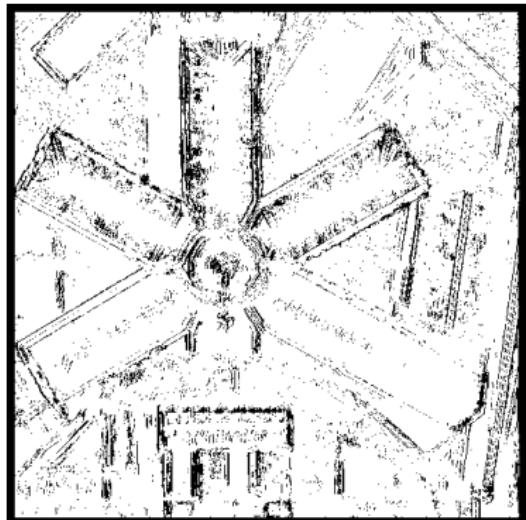
# Simulated stereo pair



# Simulated stereo pair

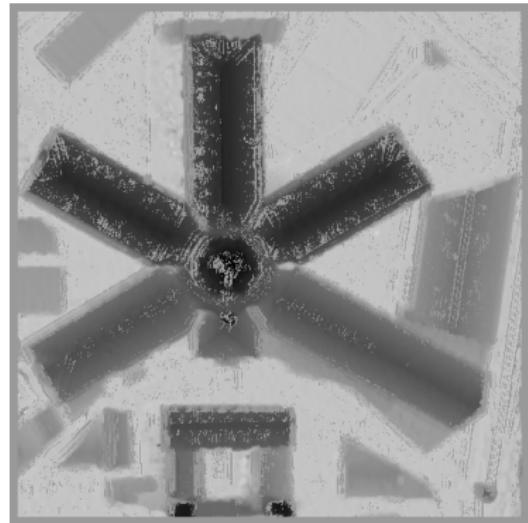


Disparity Map

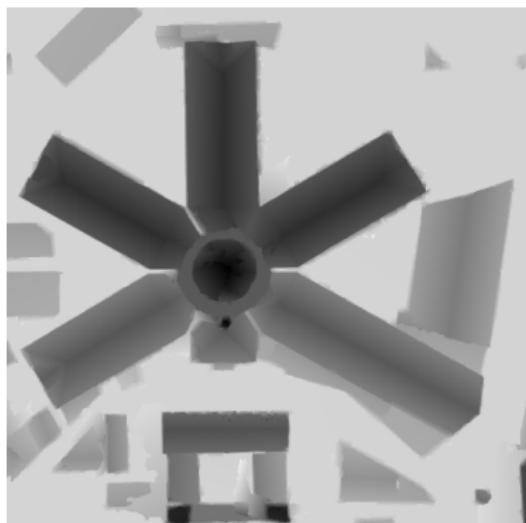


Associated mask of points

# Simulated stereo pair



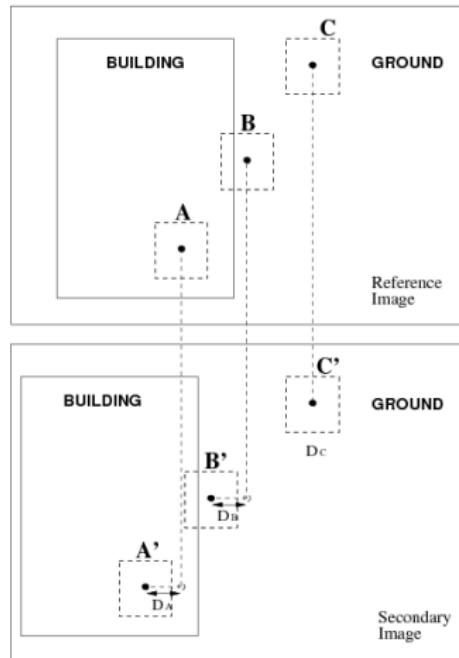
Disparity Map



Groundtruth

# Solving the adhesion problem

- Block-matching methods suffer distortions of the DEM: **Adhesion Artifact.**
- Dilation of structures due to the prevalence of borders.
- Depends on the size of the block/window.



# Solving the adhesion problem

- Let  $\varepsilon$  be the estimated disparity map between  $u$  and  $v$ .
- $\forall c \in u$ , we consider pixels in  $B_c$  minimizing

$$\alpha^c(x) = \text{Angle}\left(\frac{\nabla u}{|\nabla u|}(x), \frac{\nabla v}{|\nabla v|}(x + \varepsilon(c))\right).$$

- $\forall q \in u$  such that  $|\nabla u|(q) > 3\sigma$ , we define the **corrected disparity** as:

$$\tilde{\varepsilon}(q) = \underset{y \in B_q}{\text{Med}} \left\{ \varepsilon(y) \mid \alpha^y(q) < \underset{z \in B_y}{\text{Med}} \alpha^y(z) \right\},$$

- Points with non tolerated error:

$$M = \left\{ x \in u \mid |\varepsilon(x) - \tilde{\varepsilon}(x)| > \theta \right\}$$

# Solving the adhesion problem

- Risk points:  $\text{RP} = \bigcup_{c \in M} E_c$

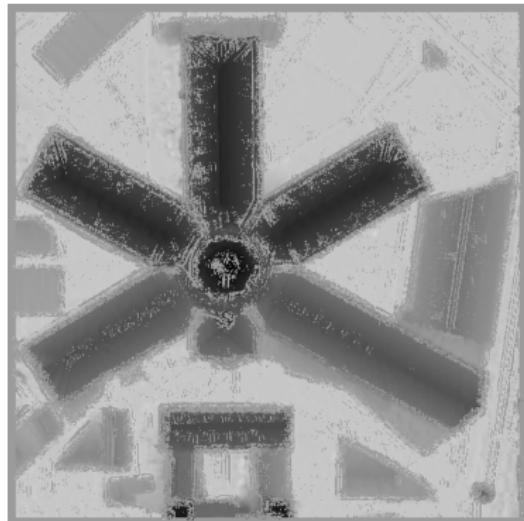
$$\forall c \in M, \quad E_c = \left\{ x \in B_c \mid |\nabla u|(x) \simeq \underset{z \in B_c}{\text{Max}} |\nabla u|(z) \right\}$$

- The final merged disparity:

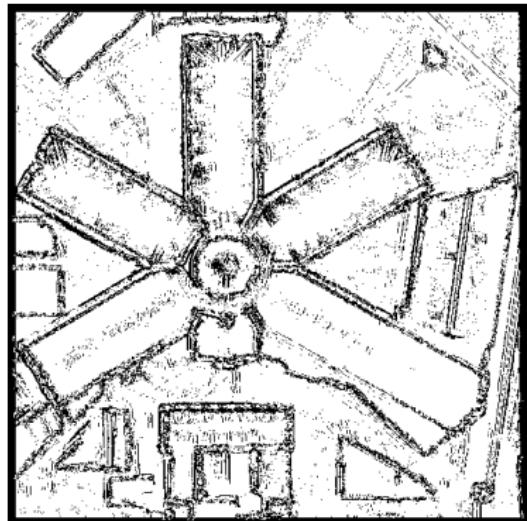
$$\varepsilon_F(x) = \begin{cases} \tilde{\varepsilon}(x) & \text{if } B_x \cap \text{RP} \neq \emptyset \\ \varepsilon(x) & \text{otherwise} \end{cases} \quad (4)$$

- Post-processing

# Final result of the simulated stereo pair

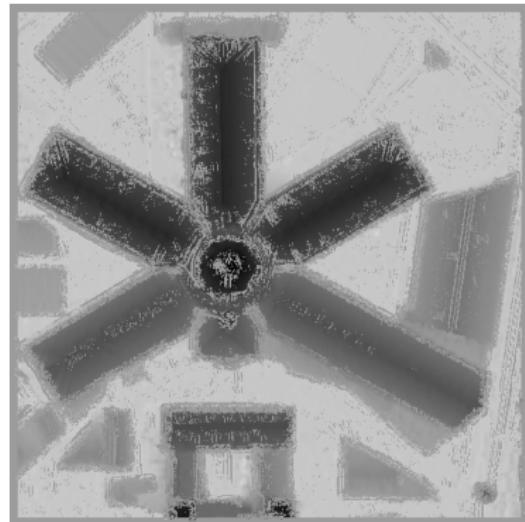


Merged Disparity Map

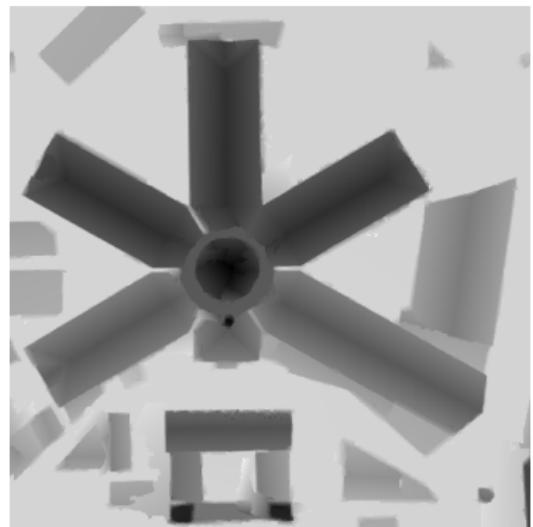


Associated mask of points

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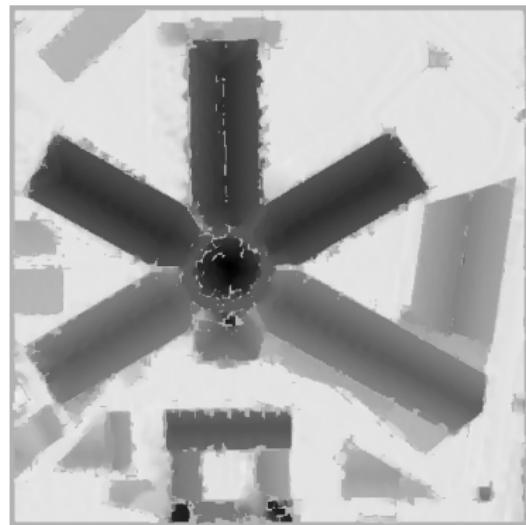


Merged Disparity Map

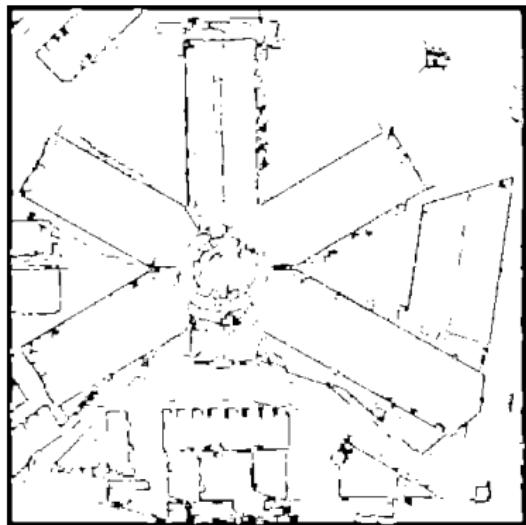


Groundtruth

# Final result of the simulated stereo pair



Post-processing



Associated mask of points

# Paire simulée

	RMSE	Matched points	Bad matches
$\varepsilon$	0.163	89.41%	0.46%
$\varepsilon_{lsd}$	0.075	45.39%	0.01%
$\varepsilon_{final}$	0.093	95.49%	0.12%

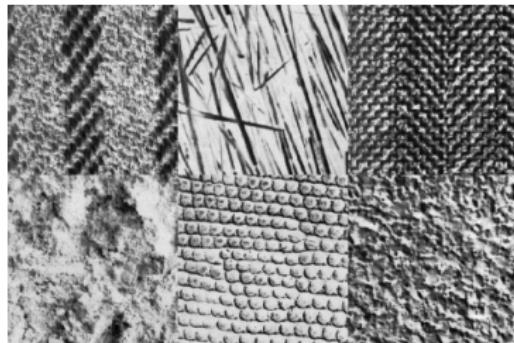
- $RMSE = \left( \frac{\sum_{i \in I} (u_i - gt_i)^2}{|I|} \right)^{\frac{1}{2}}$ ,
- Bad matches:  $|u_i - gt_i| > 1$ .

# Noise sensitivity

SNR	RMSE	Matched points	Bad matches
$\infty$	0.063	63.9	0.01
357.32	0.075	45.39	0.01
178.66	0.081	33.15	0.01
125.06	0.096	24.73	0.02

- $RMSE = \left( \frac{\sum_{i \in I} (d_i - gt_i)^2}{|I|} \right)^{\frac{1}{2}}$ ,
- $SNR = \frac{\| u \|_2}{\sigma_n}$ ,
- Bad matches:  $|d_i - gt_i| > 1$ .

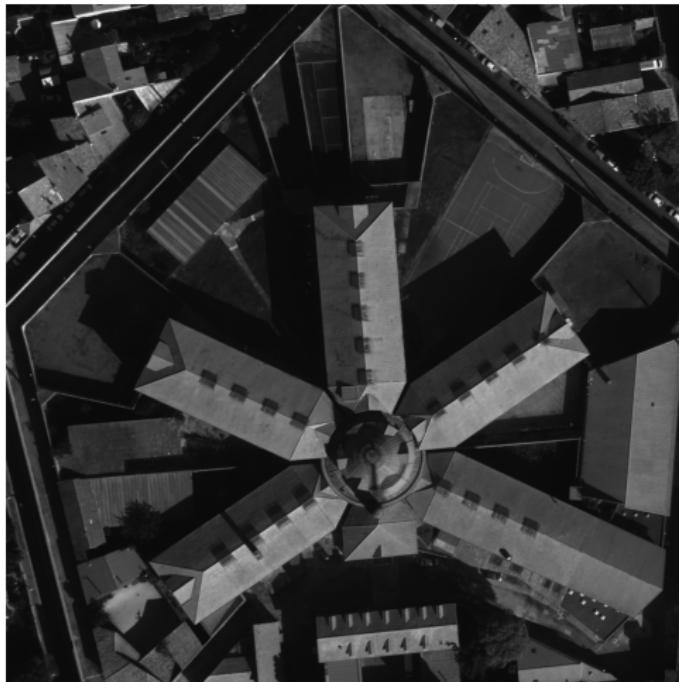
# Textured images



SNR	RMSE	Matches	Bad matches
$\infty$	0.0053	100	0.0
96.38	0.0073	99.8	0.0
48.19	0.0109	99.8	0.0
32.12	0.0160	98.7	0.0
24.09	0.0203	87.1	0.0

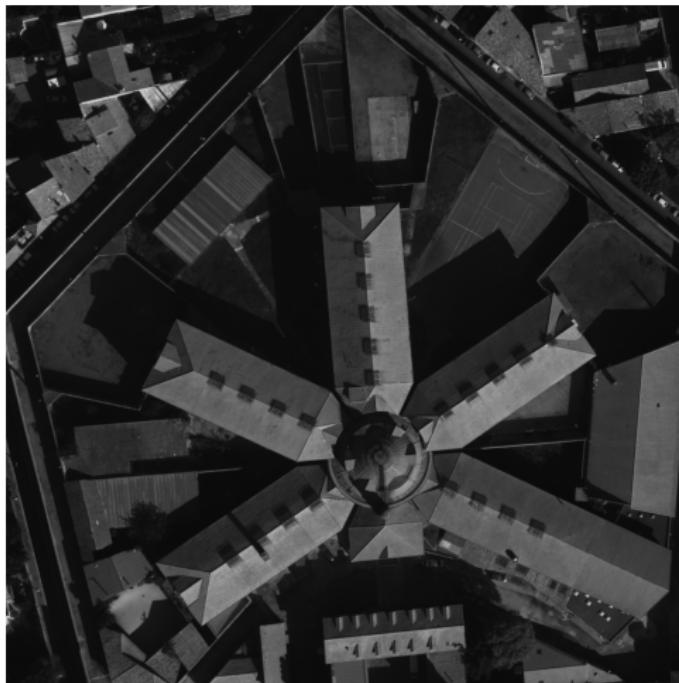
# Paire réelle

50194-60194.  $B/H = 0.04$ . Résolution = 20cm.



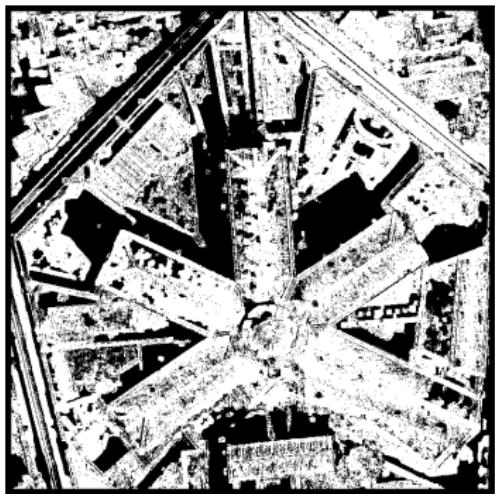
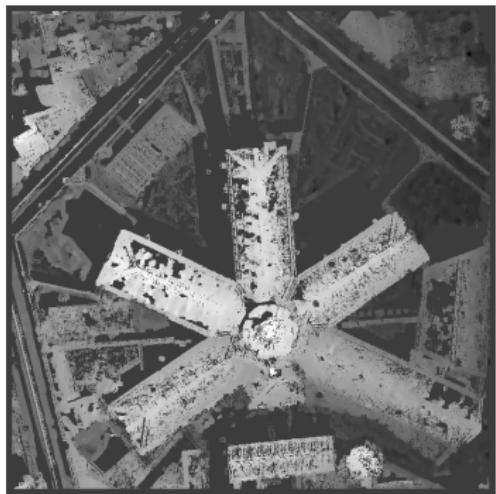
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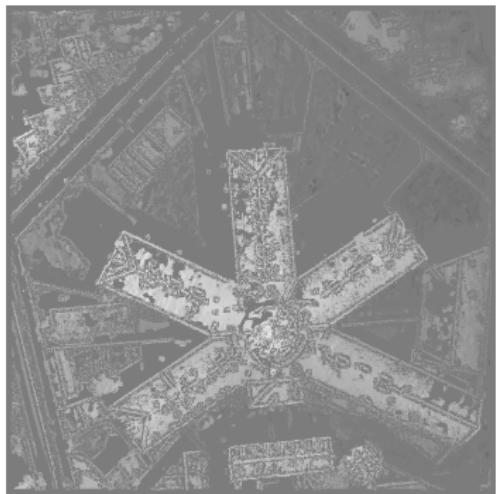
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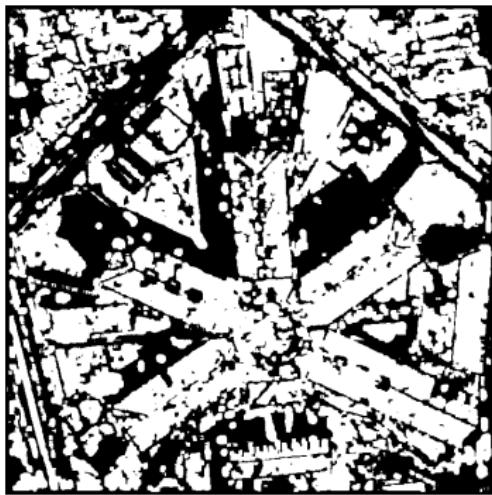
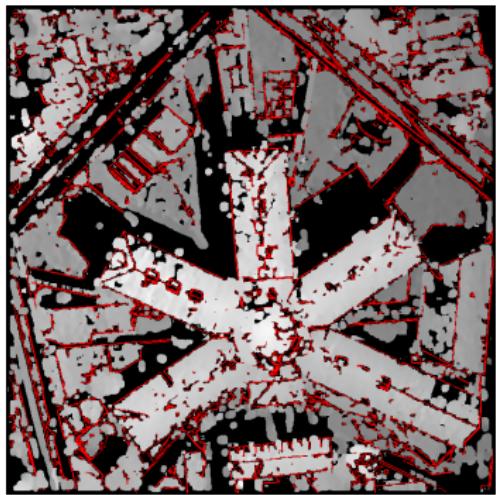
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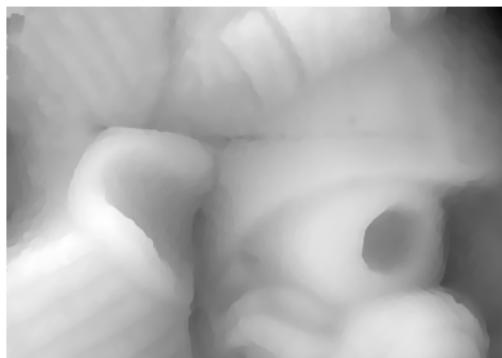


# Paire réelle

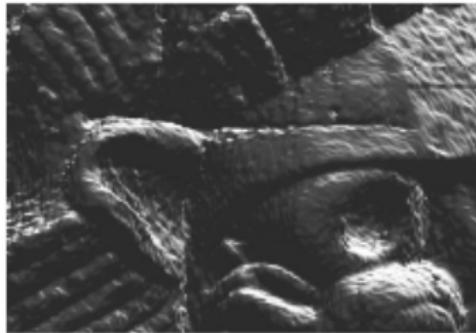
50194-60194.  $B/H = 0.04$ . Résolution = 20cm.



# Lion



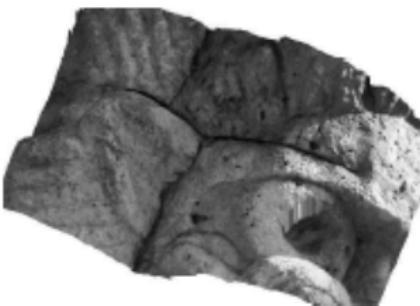
## Lion



(a) upper view of a 3D rendering of the computed surface



(b) slanted view of a 3D rendering of the computed surface



(c) slanted view of the computed surface with the reference image rendered as texture on the surface

## Conclusions

- Block-matching with nearly zero false matching rate is possible !!  
(at the cost of 50% to 80% sparse disparity map)
- Fine-localization error for these matches is between 0.1 and 0.01 pixels in typical SNR settings.

## Improvements

- Extend to color and vector-valued images

## Perspectives

- Use the resulting point cloud to automatically detect 3D structure
- Use detected 3D structure for interpolation and regularization